Miami-Dade Community College

<u>Common Course Number:</u> **MAS 3105**

Course Title: Linear Algebra

Catalogue Course Description:

This course is designed for students who are majoring in secondary mathematics education, mathematics, science, or engineering. Major topics include systems of linear equations, matrices, determinants, vector spaces, linear transformations, eigenvectors and eigenvalues, inner-product spaces and orthogonality. *Three credits.*
Prerequisite: MAC 2312.

Credit Hours Breakdown: 3 lecture hours

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Co requisite: None

Course Competencies:

Competency 1: The student will demonstrate knowledge of systems of linear equations.

Upon successful completion of this course, the student will demonstrate knowledge of systems of linear equations by:

A. Identifying the geometric nature of the solution sets of systems of linear equations in R^2 and R^3.

B. Using Gaussian elimination to determine the solution set of a given linear system.

C. Using Gauss-Jordan elimination to find the solution set of a given linear system.

D. Solving linear systems using a graphing utility.

Competency 2: The student will demonstrate knowledge of matrix algebra.

Upon successful completion of this course, the student will demonstrate knowledge of matrix algebra by:

A. Computing linear combinations of two or more matrices.

B. Computing the product of two matrices.

C. Determining the transpose of a given matrix.

D. Computing the trace of a given square matrix.

E. Expressing a given system of linear equations as a single matrix equation Ax = b.

F. Proving theorems of matrix operations.

G. Explaining the relationship between matrix row operations and elementary matrices.

H. Using elementary matrices and/or row operations to find the inverse of a matrix.

I. Identifying various properties of inverses of matrices.

J. Identifying diagonal, triangular, symmetric, and skew-symmetric matrices.

K. Using a graphing utility to perform matrix addition, scalar multiplication, and matrix multiplication; and to find the inverse and transpose of a matrix.

Competency 3: The student will demonstrate knowledge of determinants and their properties.

Upon successful completion of this course, the student will demonstrate knowledge of determinants and their properties by:

A. Using the determinant function to evaluate second- and third-order determinants.

B. Computing determinants up to order five by row reduction.

C. Finding the determinant of the inverse matrix A^{-1}, given the determinant of matrix A.

D. Evaluating determinants up to order five by cofactor expansion.

E.

Competency 4: The student will demonstrate Comprehension of the Euclidean n-space.

Upon successful completion of this course, the student will demonstrate comprehension of the Euclidean n-space by:

A. Using the basic operations (addition, scalar multiplication, Euclidean inner-product) in Euclidean n-space.

B. Proving properties of the inner product in the Euclidean n-space.

C. Computing norms and distances in a Euclidean n-space.
D. Applying properties of the norm, the distance function, and the Cauchy-Schwartz inequality.

E. Generalizing the concept of orthogonality in R^2 and R^3 to Euclidean n-space.

Competency 5: The student will demonstrate Knowledge of linear transformations from R^n to R^m.

Upon successful completion of this course, the student will demonstrate knowledge of linear transformations from R^n to R^m.

A. Defining linear transformations from R^n to R^m.

B. Defining the following linear transformations in R^2 and R^3: reflection about an axis, reflection about a plane, orthogonal projection on an axis, orthogonal projection on a plane, rotation about an axis in the plane and in the space, dilation and contraction.

C. Computing the standard matrix for a linear transformation from R^n to R^m and for a composition of two linear transformations.

D. Determining the null space of a linear transformation from R^n to R^m.

E. Determining the range of a linear transformation from R^n to R^m.

F. Determining the inverse of a one-to-one linear operator on R^n.

Competency 6: The student will demonstrate Knowledge of general vector spaces.

Upon successful completion of this course, the student will demonstrate knowledge of vector spaces by:

A. Identifying the vector space axioms.

B. Recognizing how a set of matrices, a set of vectors in R^n, and a set of polynomials, together with appropriate operations, can be considered as vector spaces.

C. Using closure properties to prove whether or not a subset W of a vector space V is a subspace of V.

D. Determining the subspace spanned by a set of vectors in a vector space V and the relation of this subspace with other subspaces containing those vectors.

E. Proving that a given set of vectors in V is linearly independent or linearly dependent.

F. Explaining that the size of a minimal spanning set of a vector space is equal to the size of a largest linearly independent subset of the vector space.

G. Finding a basis for a vector space.

H. Using the concept of dimension of a vector space to assist in finding a basis.

I. Computing the row and column spaces of matrices.

J. Using the concept of row space invariance as a tool to find bases and dependency equations.

L. Using the concept of rank of a matrix to establish the consistency of a linear system of equations.

Competency 7: The student will demonstrate knowledge of general inner-product spaces.

Upon successful completion of this course, the student will demonstrate knowledge of general inner-product spaces by:

A. Identifying examples of inner product spaces that are different from Euclidean n-space.

B. Recognizing how the inner-product induces the norm and distance function in general inner-product spaces.

C. Applying the properties of the inner product, the norm, the distance function, and the Cauchy-Schwartz inequality in an inner-product space.

D. Computing the orthogonal complement $W^{\perp}$ of a subspace W of an inner product space.

E. Explaining how the inner product establishes a geometric connection between the null-space and the row space of a matrix.

F. Constructing an orthonormal basis for an inner product space by using the Gram-Schmidt process.

G. Defining an orthogonal matrix.

Competency 8: The student will demonstrate an understanding of eigenvalues and eigenvectors.

Upon successful completion of this course, the student will demonstrate knowledge of eigenvalues and eigenvestors by:

A. Computing the eigenvalues of a given matrix and determining the corresponding eigenvectors.

B. Determining bases for the eigenspaces of a matrix.

C. Defining similar matrices.

D. Using eigenvalues to determine whether a given matrix or linear operator is diagonalizable.

E. Using eigenvalues to find an orthogonal diagonal matrix similar to a given symmetric matrix.

Competency 9: The student will demonstrate knowledge of general linear transformations

Upon successful completion of this course, the student will demonstrate knowledge of general linear transformations by:

A. Determining whether a transformation from one vector space V into another vector space W is linear.

B. Determining the general rule for a linear transformation, given its behavior on a basis of the domain space.

C. Finding the standard matrix of a linear transformation.

D. Proving that the kernel and the range of a general linear transformation are subspaces of the corresponding spaces.

E. Explaining how the dimension theory for general linear transformations can be used for finding subspaces associated with them.

F. Determining whether a given linear transformation is invertible, and, if so, finding the inverse.