Miami-Dade Community College
MAS 3301 – Algebraic Structures

Course Description: This course is designed for students who are majoring in secondary mathematics education, mathematics, science or engineering. Topics include set theory, basic properties of the integers, groups, rings, fields and the homomorphisms of these algebraic structures. (3 credits.)

Prerequisite: MAC 2312.

Course Competencies:

Competency 1: The student will demonstrate an understanding of set theory by

a) performing the following set operations: union, intersection, difference and complementation.
b) applying the following properties of set operations: associative, distributive and DeMorgan's Laws.
c) determining the domain, codomain, and range of a given mapping between two sets.
d) given a mapping between two sets, determining the image of a given subset of the domain and/or the inverse image of a given subset of the range.
e) given a mapping between two sets, determining whether or not the mapping is injective, surjective or bijective.
f) determining whether a given relation on a set is an equivalence relation.
g) identifying the equivalence classes of a given equivalence relation.
h) proving set-theoretical properties.

Competency 2: The student will demonstrate an understanding of the basic properties of the integers ($\mathbb{Z}$) by

a) applying the principle of mathematical induction.
b) determining the greatest common divisor (gcd) and the least common multiple (lcm) of two integers using prime factorizations.
c) determining the gcd of two integers using the Euclidean Algorithm.
d) applying the division algorithm to compute the quotient and remainder in the division of two integers.
e) using the gcd to determine if two given integers are relatively prime.
f) performing addition and multiplication and computing inverses in the set $\mathbb{Z}_n$, the set of congruence classes modulo n.
g) solving linear congruences.
h) proving properties of $\mathbb{Z}$ or $\mathbb{Z}_n$.

Competency 3: The student will demonstrate an understanding of groups by

a) identifying the set of axioms that define the algebraic structure of a group.
b) providing examples of groups, such as ($\mathbb{Z}$, +), ($\mathbb{Z}_n$, +), ($\mathbb{Z}_p^*$, ·), where p is prime, (S_n, ○), ($M_{2x2}(\mathbb{R})$, ·), etc.
c) determining whether a particular group is Abelian or non-Abelian.
d) using the multiplication table of a given finite group to perform operations in the group.
e) proving whether or not a given subset of a group is a subgroup.
f) determining whether a group is cyclic.
g) recognizing that a cyclic group of order n is isomorphic to the group $\mathbb{Z}_n$, and a cyclic group of infinite order is isomorphic to $\mathbb{Z}$.
h) identifying cycles as elements of the symmetric group S_n.
i) identifying the alternating subgroup, A_n, of the symmetric group S_n.
j) decomposing an element of S_n into a product of transpositions or disjoint cycles.
k) recognizing that every group is isomorphic to a group of permutations (Cayley's Theorem).
l) identifying mappings between groups ($f : G \rightarrow H$) that are homomorphisms.
m) determining whether or not a given subgroup of a group G is a normal subgroup, specifically, the kernel of a homomorphism.
n) constructing quotient groups of a group G.
o) recognizing that the order of any subgroup of a group G must divide the order of G (Lagrange's Theorem).
p) proving properties of groups.

Competency 4: The student will demonstrate an understanding of rings and associated structures by

a) identifying the set of axioms that define the algebraic structure of a ring.
b) proving whether or not a given subset of a ring is a subring.
c) determining whether or not a particular ring has zero divisors.
d) classifying particular rings as integral domains, division rings, fields, rings with unity, commutative rings, and non-commutative rings, and verifying these classifications.
e) identifying the properties that determine that a mapping between rings is a homomorphism.
f) proving whether or not a given mapping between two rings is a homomorphism.
g) proving properties of rings.