Miami-Dade Community College
MAS 4203- Number Theory

Course Description: Topics include mathematical induction, divisibility, the Euclidean algorithm, primes, the Fundamental Theorem of Arithmetic, number-theoretic functions, congruence, linear Diophantine equations, linear congruences, the Chinese Remainder Theorem, and the theorems of Euler, Fermat, and Wilson. *Three credits*.

Prerequisite: MAC 2312.

Course Competencies:

Competency 1: The student will demonstrate an understanding of the basic properties of the integers by

a) applying the principle of mathematical induction.
b) finding divisors of a given integer
c) finding the greatest common divisor (gcd) of two integers using the Euclidean Algorithm.
d) applying the Pythagorean Triples Theorem to generate sets of Pythagorean triples.
e) applying the division algorithm to compute the quotient and remainder in the division of two integers.
f) proving basic properties of the integers.

Competency 2: The student will demonstrate an understanding of primes and relatively prime integers by

a) identifying prime numbers by the sieve of Eratosthenes.
b) finding the unique prime factorization of a given integer.
c) determining the gcd and the lcm of two integers using prime factorizations.
d) determining whether or not two integers are relatively prime.
e) expressing 1 as a linear combination of two given relatively prime integers.
f) approximating $\pi(x)$, the number of primes less than or equal to a given real number x, using the Prime Number Theorem.
g) determining whether a given prime is a Mersenne prime.
h) generating perfect numbers using Euclid's Perfect Number Formula.

Competency 3: The student will demonstrate an understanding of linear congruences by

a) solving a linear congruence.
b) using the Chinese Remainder Theorem to solve a system of two or more simultaneous linear congruences.
c) performing operations among members of a given complete residue system.
d) determining whether or not a given integer has a multiplicative inverse, mod n, and, if so, finding the inverse.
e) applying Fermat's little theorem to establish congruences with a prime modulus.
f) applying Wilson's theorem to establish congruences with a prime modulus.

Competency 4: The student will demonstrate an understanding of number-theoretic functions by

a) computing values of Euler's phi function, $\Phi(m)$
b) establishing properties of Euler's phi function.
c) performing numerical computations with the Euler's phi function.
d) applying Euler's theorem to reduce large powers modulo n.
e) computing values of the sigma function, $\sigma(n)$.
f) establishing properties of the sigma function.
g) performing numerical computations using the sigma function.

Competency 5: The student will demonstrate an understanding of Diophantine equations by

a) solving a linear Diophantine equation ($ax + by = c$).
b) solving a Pythagorean triples equation ($X^2 + Y^2 = Z^2$).
c) solving Pell's equations ($x^2 - Dy^2 = 1,$ where D is a fixed positive integer that is not a perfect square).