

## The Magic Math

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**Abstract:** The content of this article provides quick and easy methods to make calculations involving real numbers. These tricks involve the Chinese chopsticks method of multiplication, the multiplication table of 9, the squares of 5, multiplication without calculator, and the Pythagorean Theorem. These tricks should be very interesting for students taking mathematic classes such as MAT0002 (basic arithmetics), MAT0020 (elementary algebra), MAT1033 (intermediate algebra), MAC1114 (trigonometry), and MAC1147 (pre-calculus), as they reveal to be fast, reliable and time saving.

- **Introduction: In order to inspire students that math is not boring but instead can be very interesting, this article demonstrates few examples to show how math can be beautiful and intellectually challenging.**

### Chinese chopsticks method for the multiplication of numbers without a table of multiplication

Once upon a time there was a Chinese farmer who never went to school; therefore he did not learn how use his abacus. To sell his goods in the market, he used basic counting of chopsticks. To help him know how much  $23 \times 12$  is, we are going to use Chinese chopsticks to solve the multiplication

#### 1) Regular case

$$23 \times 12 = ?$$

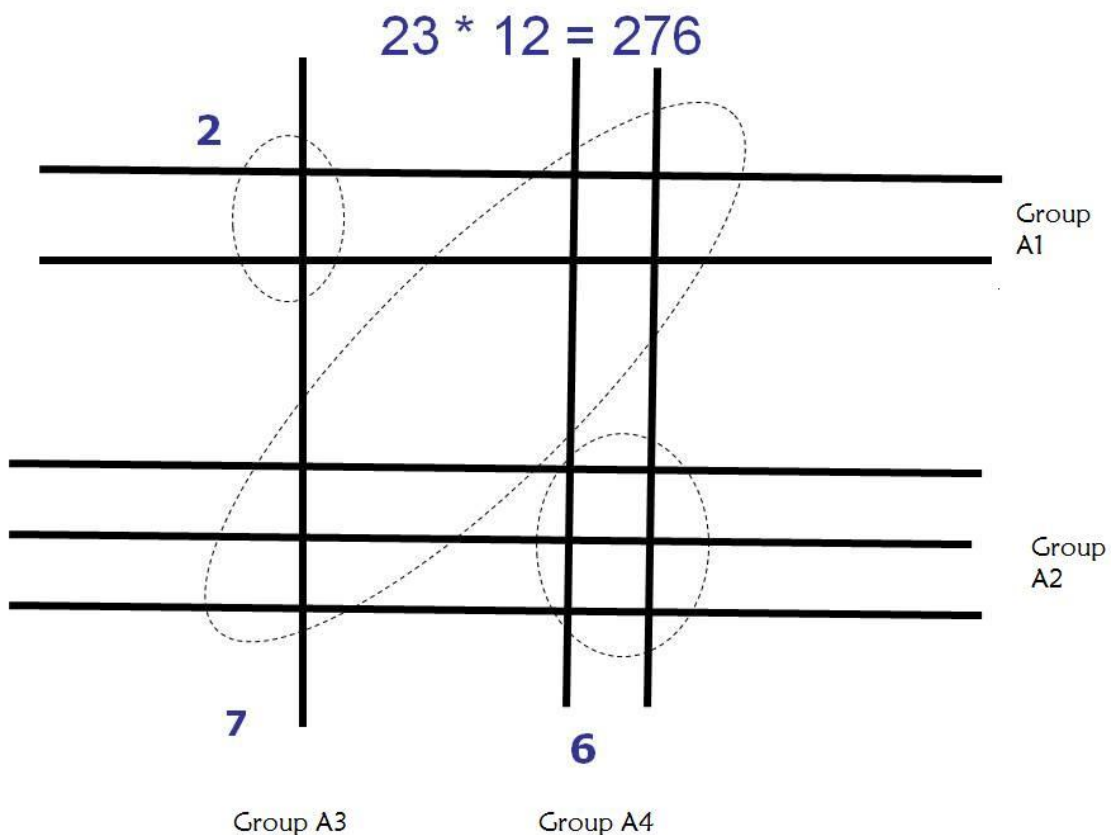
In this method, each number forms a group, and each group contains a number of chopsticks equal to the number of the group they represent. Let's call group one  $A_1 = 2$  chopsticks, group two  $A_2 = 3$  chopsticks, group three  $A_3 = 1$  chopstick and finally group four  $A_4 = 2$  chopsticks. Each group is arranged in the following order:  $A_1$  in the top of the page,  $A_2$  in the bottom,  $A_3$  to the left, and  $A_4$  to the right. Each group has to be disposed in a way that the chopsticks from group  $A_1$  and  $A_2$  can be distinguished to be from two different groups;  $A_3$  and  $A_4$  should be placed over  $A_1$  and  $A_2$  at a  $90^\circ$  angle, but also clearly distinguishable. Because of the way they are placed,  $A_3$  and  $A_4$  must have points of intersection with  $A_1$  and  $A_2$ ; the entire method relies on these intersection points, and throughout this article, we will designate them with the + symbol.

$A_1 + A_3$  is in the top left corner,  $A_1 + A_4$  in the top right corner,  $A_2 + A_3$  are in the bottom left corner and  $A_2 + A_4$  in the bottom right corner. Each intersection has several points of contacts. For example if  $A_1$  and  $A_4$  are put one on top of each other at a 90 degrees angle, they will have

four points of contact, as  $A_1$  has 2 chopsticks and  $A_4$  has also 2 chopsticks. In that same spirit,  $A_1 + A_3$  have 2 points of contact,  $A_2 + A_3$  have 3 and  $A_2 + A_4$  have 6. However, each intersection is not counted as a single entity.  $A_2 + A_3$  and  $A_1 + A_4$ , which are in the same diagonal, must be counted together; their total amount of intersection points will thus be 7.

We finally have the result for our multiplication: the three single digit numbers 2, 6 and 7. However we should arrange them accordingly to find out the final answer. The order is simple: first we write the result from  $A_1 + A_3$  which is 2, then the result for the diagonal  $A_2 + A_3$  and  $A_1 + A_4$  which is 7, and then the result from  $A_2 + A_4$  which is 6. The result is thus 276! It can actually be read from the indicated “bubbles” in the graph from left to right.

$$23 \times 12 = 276 \checkmark$$

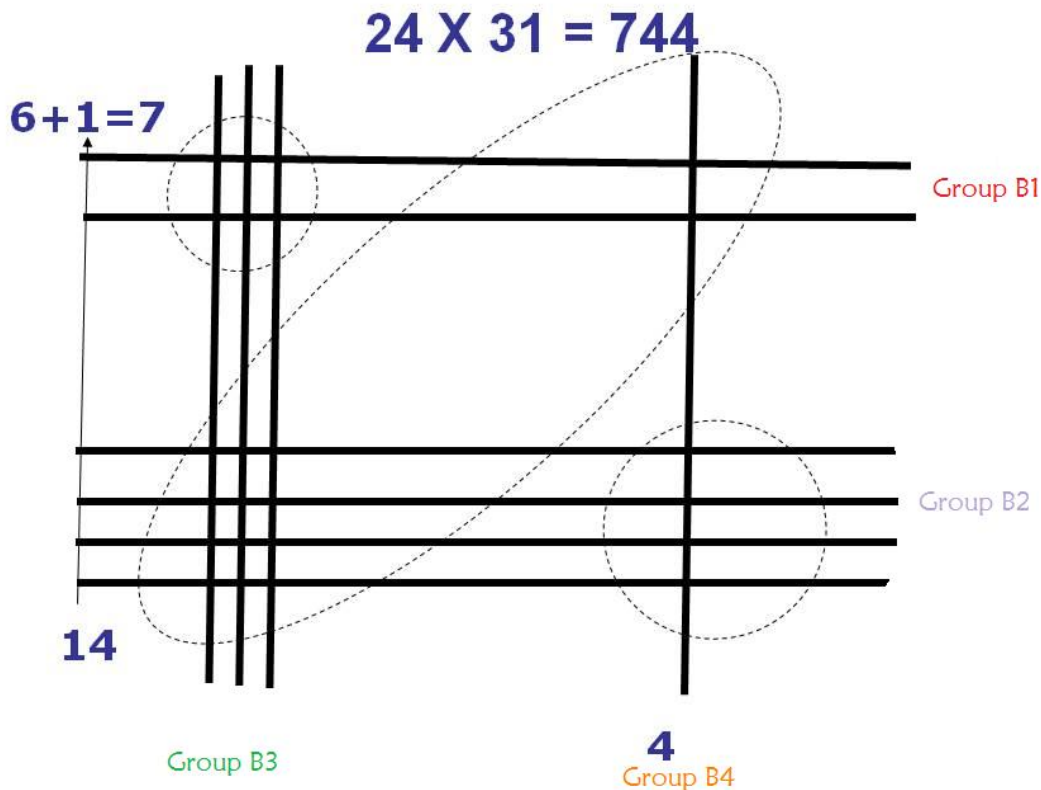


## 2) Digit carry-over case

What would happen if the numbers we have to multiply bring about intersections that have a number of touching points with two digits? To solve this, we use the digit carry-over method. Let's multiply  $24 \times 31 = ?$

This time we will use  $B_1=2$  chopsticks,  $B_2=4$  chopsticks,  $B_3=3$  chopsticks and  $B_4=1$  chopstick. We arrange them the same way as we did previously. However, this time we do not count the intersection points in any order. We should start with the bottom right corner or the  $B_2 + B_4$  intersection, followed by the diagonal formed with  $B_1 + B_4$  and  $B_2 + B_3$  and then finish counting with the top left corner or  $B_1 + B_3$  intersection. From this we obtain that  $B_2 + B_4$  has 4 points of contact, and the diagonal  $B_1 + B_4$  and  $B_2 + B_3$  has 14 points of contact. Here we pause a moment to explain the necessity of the digit carry-over case. Since each intersection should give a single digit number of points of contact, from the number 14, we take the four as being the result for the intersection and the remainder 1 is carried-over and added to the counting of the next intersection, in this case, the upper left corner or  $B_1 + B_3$  intersection. Therefore we no longer have 6 as a result of the intersection we normally would, but seven. We thus have now the result for our multiplication: 7, 4 and 4 which is correct.

$$24 \times 31 = 744 \checkmark$$



### The Magic Squares of 5

Calculating squares of numbers is fairly easy as long as we are taking the square of the numbers between one and ten. When the square involves greater numbers it becomes a little bit more complicated. However, there is a way to quickly calculate the square of numbers whose last digit is 5.

As we know,  $5^2 = 25$ . Let's rewrite the single digit 5 as the two digit number 05; its square root is still going to be 25. To investigate the squares of higher numbers, let's analyze  $25^2 = 225$ . The squares of these numbers can be found in a very easy way. We separate the number to be squares into two components: B which is the last digit of the number and A the remainder. As an example, the number 05 would be separated into  $A = 0$  and  $B = 5$ , 25 would be separated into  $A = 2$  and  $B = 5$ . Note that since the numbers we are dealing with are those ending with 5, B will always be equal to 5.

The calculation of squares is as followed:  $A \times (A+1) = C$  and  $B^2 = D$  will produce CD. Illustrating this in  $05^2$   $A = 0$  therefore  $A+1 = 1$  making  $A \times (A+1) = 0 \times 1 = 0$ . Then we have  $B^2 = 5^2 = 25$ . Finally we get  $C = 0$  and  $D = 25$  therefore  $CD = 025$  or commonly written as 25. In the same manner, if we want to square the number 25, we separate it into two components:  $A = 2$  and  $B = 5$ .  $B^2 = 25$  and  $A \times (A + 1) = 2 \times 3 = 6$ . Therefore  $C = 6$  and  $D = 25$ , making  $CD = 625$  which is the result of  $25^2$ .

$$\begin{aligned}
 5^2 &= 25 \\
 15^2 &= 225 \\
 25^2 &= 625 \\
 35^2 &= 1225 \\
 75^2 &= 5625 \\
 105^2 &= 11025
 \end{aligned}$$

### The Interesting Number 9

This case does not make any calculation easier but it is interesting to discover the nice properties about the multiplication table of the number 9.

The number 9 has a multiplication table that is totally symmetrical. The multiples of 9 from one to five are respectively 9, 18, 27, 36 and 45. It appears that the multiples of 9 from six to ten are 54, 63, 72, 81, and 90 respectively, which are the symmetric opposite of the first five numbers 09,18,27,36 and 45.

Also if we look at the table on a column, from top to bottom, we observe that the multiples of 9 from 1 to 10 are all made of both numbers from 0 to 9 as their first digit and 9 to 0 as their second digit, showing again a geometrical symmetry.

|                    |  |
|--------------------|--|
| <b>9 * 1 = 09</b>  |  |
| <b>9 * 2 = 18</b>  |  |
| <b>9 * 3 = 27</b>  |  |
| <b>9 * 4 = 36</b>  |  |
| <b>9 * 5 = 45</b>  |  |
| <b>9 * 6 = 54</b>  |  |
| <b>9 * 7 = 63</b>  |  |
| <b>9 * 8 = 72</b>  |  |
| <b>9 * 9 = 81</b>  |  |
| <b>9 * 10 = 90</b> |  |

### Interesting Application of Multiplication

In elementary school, students learn the tables of multiplication. Unfortunately, due to the constant use of calculators they stop using them; and instead of multiplying, say 4x6 in their minds, they plug it into the calculator to obtain the answer. Although using the calculator may at times prove to be a faster method to get to the mathematical answer of a problem, it deprives the students from learning some interesting ideas that can be unveiled from the tables of multiplication. One such instance is the table of two, and its concept of doubling. The doubling effect is better explained by the following example,

As you walk along a lake, you noticed that lake is inhabited by just one lotus flower. The next day, you walked by the lake, and noticed that there are two lotus flowers. The following day, you see four lotus flowers instead of two. If you know the lake will be completely covered on the 30<sup>th</sup> day, how long would it take for half of the lake to be

covered by lotus flowers? Keep in mind that you do not know the size of the lake, nor the size of the lotus flower.

You can start this problem by noticing that as each day goes by the amount of lotus flowers is doubled. Then you designate the initial lotus flower as  $x$ . Since the amount of flowers is doubled everyday, you would multiply it by two every time. This means that on day one you have  $2x$ . On the second day you have  $2(2x) = 2^2x$ . And so on and so forth. If the doubling concept is kept in mind, then it can be said that by the 29<sup>th</sup> day, the lake should be half occupied, and so the next (the 30<sup>th</sup>), after it is doubled, the lake would be completely covered.

### Another Way of Multiplying Without Using a Calculator

Really early in school life, students adopt the use of calculators in a very addictive way. Many times they cannot solve the simplest calculation without plugging it in a calculator. There are many ways to perform calculations without using a calculator; however they can be tedious at times. Here we will be demonstrating a very simple and easy way to calculate squares of numbers that are not multiples of five.

The steps to follow are simple: If you have a number you want to square, let's call it  $x^2$ . Since  $x^2 = x^2 - y^2 + y^2$ , and  $x^2 - y^2 = (x + y)(x - y)$ , therefore we obtain  $(x + y)(x - y) + y^2$ , where  $x$  is the number to be squared, and  $y$  is the difference between that number and the next perfect square. This is the formula we are going to use to compute our squares without the calculator!

**Example:**

$$\begin{aligned} 97^2 &= (97 + 3)(97 - 3) + 3^2 \\ &= 100 * (94) + 9 \\ &= 9400 + 9 = 9409 \end{aligned}$$

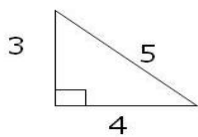
$$\begin{aligned} \text{Prove: } x^2 &= x^2 - y^2 + y^2 \\ &= (x + y)(x - y) + y^2 \end{aligned}$$

### The Pythagorean Theorem

All students who have gone through Algebra, Geometry and Trigonometry, know about the Pythagorean Theorem, as it is one of the first mathematical theorems that is taught. It states that for a right triangle, the addition of the squares of the two shorter sides of the triangle is equal to the hypotenuse (the longest side) of that same triangle. It is expressed by the equation  $A^2 + B^2 = C^2$ .

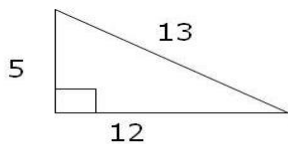
The most common and well known Pythagorean triangles are the 3-4-5, the 5-12-13, and the 7-24-25 triangles. Looking at them, we can discover a pattern. For all three of them the shortest sides are equal to 3, 5 and 7 respectively. They are all arithmetic sequences with a difference of 2. Also, another pattern that we can notice is that to find the value of the second shorter side of the triangle. By multiplying the shortest with a sequenced number and then adding it to the result:  $(3 \times 1) + 1 = 4$ ;  $(5 \times 2) + 2 = 12$ ;  $(7 \times 3) + 3 = 24$ . Finally, the hypotenuse is obtained by adding 1 to the previously found side:  $4 + 1 = 5$ ;  $12 + 1 = 13$ ;  $24 + 1 = 25$ .

We can, therefore summarize our patterns into a formula. Whenever the shortest side of the right triangle is an odd number, the second shorter side is found by first finding the component  $y$  that will be plugged in later into the formula:  $xy + y = B$ .  $x$  is the starting number in the arithmetic sequence,  $y$  is a multiplier that increases by 1 for each calculation. This calculation produces the number for the hypotenuse.



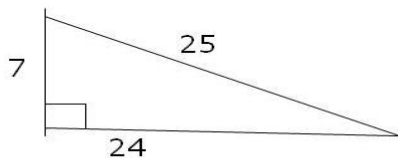
$$3 \times 1 + 1 = 4$$

$$4 + 1 = 5$$



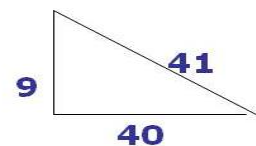
$$5 \times 2 + 2 = 12$$

$$12 + 1 = 13$$



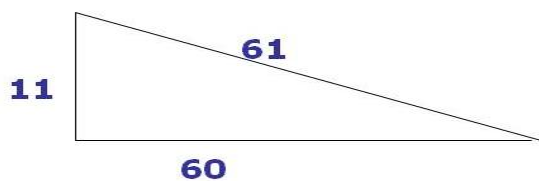
$$7 \times 3 + 3 = 24$$

$$24 + 1 = 25$$



$$9 \times 4 + 4 = 40$$

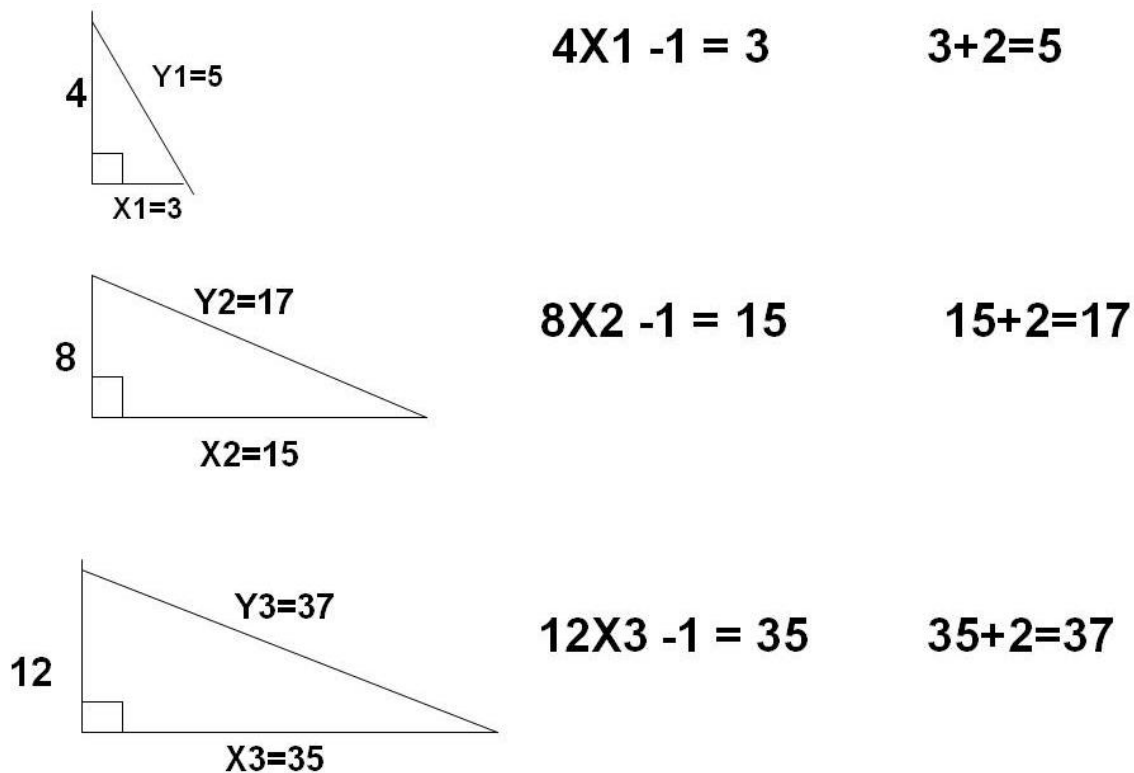
$$40 + 1 = 41$$



$$11 \times 5 + 5 = 60$$

$$60 + 1 = 61$$

What happens if the shortest side of the Pythagorean triangle is an even number? Well, the answer is simple. We follow steps similar to the prior ones, having two corrections to them. Whenever the shortest side is an even number, the second shorter side is computed with the formula  $(xz - 1)$  where  $z$  as a multiplier that starts at 1 and decreases by 1 for next calculation. When  $z_1=1$ ,  $x_1=4$ ; when  $z_2=2$ ,  $x_2=8$ . We then find the regularity of the right triangle starting with even numbers as being 4. We can predict any number down the road for the second shorter side to be the previous + 4. The hypotenuse is then found by adding 2 to the value of the second shorter side of the triangle.



### The Magic Pascal Triangle

The Pascal triangle was created by the French mathematician Pascal in the mid seventeenth century. Looking at it, it resembles a simple triangle with numbers inside. But in reality, it is a beautifully intriguing multiplication table. Within the triangle, there are many interesting characteristics we can discover.

- a) Symmetry in the diagonal direction:  
Each diagonal of the triangle is formed by a set of consecutive numbers that have a symmetric image on the other side of the triangle
- b) Symmetry in the horizontal direction:

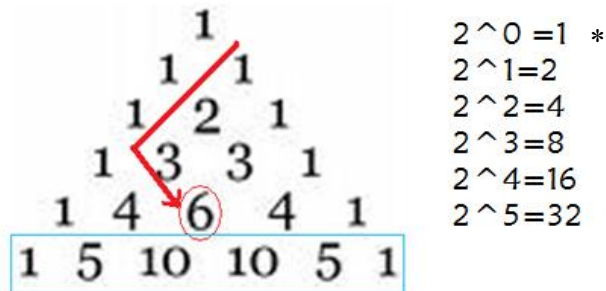
Imagine the triangle is vertically separated in half. Each horizontal line from one side has its symmetric image on the other side of the triangle. Take, for example, the fifth horizontal line of our graph, if you cut it in half right through the middle, you observe the same pattern of numbers to be exactly symmetric: 1-5-10 is the exact reflection of 10-5-1.

- c) Multiples of two in the horizontal direction:

If we add all the numbers of each horizontal line of the triangle, their results are equal to exponentials of 2 in the perfect order. For example, the sum of the first row is 1 and  $2^0 = 1$ ; the sum of the second row is 2 and  $2^1 = 2$ , and the sum of the third row is 4 and  $2^2 = 4$ ; this pattern goes on indefinitely.

- d) Addition in L shape:

The fourth characteristic of the Pascal triangle resides in the addition of numbers forming L-shapes. If we add any series of numbers in a diagonal line, starting with the most outer number in that line, their total is equal to the number forming the L arm. To illustrate that fact, let's take for example the second diagonal from our graph; the series starts with the most outer number 1 and if we add  $1+2+3=6$  which is the number forming the L with these three numbers.



\* Note:  $2^x$  means  $2^x$

### Shortcut to Obtain Partial Fractions

For students, the operation to calculate partial fractions can be tedious at times. Here we will present the Heaviside method in three shortcut cases where calculating partial fractions is not that hard. These methods can be used for College Algebra, Calculus or Differential Equations.

#### Case A

Let's find the partial fractions for  $F(s) = \frac{\text{---}}{(\text{---})(\text{---})}$

$$F(s) = \frac{\text{---}}{(\text{---})(\text{---})} = \text{---} + \text{---}$$

$$\text{Let } \frac{1}{(s-2)(s+4)} = ? \left( \frac{\quad}{s-2} + \frac{\quad}{s+4} \right)$$

Let's find ? = -

$$\text{Replace } x = s^2$$

$$\frac{1}{(s-2)(s+4)} = \frac{1}{6} \left( \frac{1}{s-2} - \frac{1}{s+4} \right)$$

Multiply by (3s+1)

$$F(s) = -\frac{1}{6(s-2)} + \frac{1}{6(s+4)} \quad \checkmark$$

### Case B

$$\text{Let's find the partial fractions for } F(s) = \frac{(s-1)}{(s-1)^2 + 1}$$

$$\text{Let } F(s) = \frac{(s-1)}{(s-1)^2 + 1} = \frac{A}{s-1} + \frac{B}{s-1} + \frac{C}{s+1} \quad [\text{This is required for solving inverse Laplace Transforms}]$$

$$F(s) = \frac{(s-1)}{(s-1)^2 + 1}$$

$$1 - s(4 - 3s) = A((s-1)^2 + 1) + B(s-1) + sC$$

\*The cover-up technique requires the effective elimination of more coefficients by letting  $s=0$

$$1 = 2A \rightarrow A = \frac{1}{2}$$

$$\begin{aligned} \text{Let } s = 1 \rightarrow 1 - 1(4 - 3) &= A + C \\ 0 &= A + C \\ C &= -A = -\frac{1}{2} \end{aligned}$$

$$\begin{aligned} \text{Let } s = -1 \rightarrow 1 + 1(4 + 3) &= 5A + B(-2) + (-1)C \\ 8 &= 5A + 2B - C \\ 2B &= 8 - 5A + C \\ &= 8 - \frac{5}{2} - \frac{1}{2} \\ &= 5 \\ B &= \frac{5}{2} \end{aligned}$$

$$F(s) = \frac{1}{2(s-1)} + \frac{5}{2(s-1)} - \frac{1}{2(s+1)}$$

$$F(s) = - + \frac{\quad}{(\quad)} - \frac{\quad}{((\quad) \quad)} \checkmark$$

**Case C**

Let's find the partial fraction for  $F(s) = \frac{\quad}{(\quad)}$

$$\text{Let } F(s) = \frac{\quad}{(\quad)} = \frac{\quad}{(\quad)} + \frac{\quad}{(\quad)} + \frac{\quad}{(\quad)}$$

We modify the numerators by matching them with (s-2)

$$\begin{aligned} s^2 + 4s - 5 &= ((s-2) + 2)^2 + 4((s-2) + 2) - 5 \\ &= (s-2)^2 + 4 + 4(s-2) + 4(s-2) + 8 - 5 \\ &= (s-2)^2 + 8(s-2) + 7 \end{aligned}$$

Therefore

$$F(s) = \frac{(\quad) \quad (\quad)}{(\quad)}$$

$$F(s) = - + \frac{\quad}{(\quad)} + \frac{\quad}{(\quad)} \checkmark$$

REF: "Joy of Math" by Arthur Benjamin

Biography: