Classroom Assessment Techniques and Their Implementation in a Mathematics Class

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ABSTRACT

Classroom assessment is one of the most significant teaching strategies. It is a major component of classroom research at present. Classroom Assessment Techniques (CAT's) are designed to help teachers measure the effectiveness of their teaching by finding out what students are learning in the classroom and how well they are learning it. This paper deals with the implementation of Classroom Assessment Techniques, namely, “Course-Related Self-Confidence Surveys,” “Muddiest Point,” and “Exam Evaluations,” in a Business Calculus Class. These techniques are used for assessing:

(i) Course-Related Knowledge and Skills;
(ii) Learner Attitudes, Values, and Self-Awareness;
(iii) Learner Reactions to Instruction.

Theme: Educational Research

Keywords: Attitudes, Assessment Technique, Exam Evaluations, Muddiest Point, Self-Confidence
1. Introduction

There are two fundamental issues with which the educational reformers are concerned. These are as follows: (i) The students’ learning in the classroom; and (ii) The effectiveness of the teaching by teachers in the classroom. To answer these questions, the movement for Classroom Research and Assessment was initiated during the 1990's by Thomas A. Angelo and K. Patricia Cross, who devised various Classroom Assessment Techniques (known as CAT’s), (see, for examples, Angelo and Cross (1993), among others, for details). They developed these CAT’s in order to help teachers to measure the effectiveness of their teaching by finding out what students are learning in the classroom and how well they are learning. According to Angelo and Cross (1993), “These CAT’s are designed to encourage college teachers to become more systematic and sensitive observers of learning as it takes place everyday in their classrooms. Faculties have an exceptional opportunity to use their classrooms as laboratories for the study of learning and through such study to develop a better understanding of the learning process and the impact of their teaching upon it.” Thus, in Classroom Assessment Approach, students and teachers are involved in the continuous monitoring of students’ learning. It gives students the feedback of their progress as learners. The faculties, on the other hand, get to know about their effectiveness as teachers. According to Angelo and Cross (1993), the founders of classroom assessment movement, “Classroom Assessments are created, administered, and analyzed by teachers themselves on questions of teaching and learning that are important to them, the likelihood that instructors will apply the results of the assessment to their own teaching is greatly enhanced.” Following Angelo and Cross (1993), some important characteristics of Classroom Assessment Approach are given below:

(i) LEARNER-CENTERED
(ii) TEACHER-DIRECTED
(iii) MUTUALLY BENEFICIAL
(iv) FORMATIVE
(v) CONTEXT-SPECIFIC
(vi) ONGOING
(vii) ROOTED IN GOOD TEACHING PRACTICE

According to a report by the Study Group on the Conditions of Excellence in American Higher Education (1984), “There is now a good deal of research evidence to suggest that the more time and effort students invest in the learning process and the more intensely they engage in their own education, the greater will be their satisfaction with their educational experience, and their persistence in college, and the more likely they are to continue their learning” (p. 17). As observed by Angelo and Cross (1993), “Active engagement in higher learning implies and requires self-awareness and self-direction,” which is defined as “metacognition” by cognitive psychologists. According to Weinstein and Mayer (1986), the following are the four activities that help students become more efficient and effective learners:

(i) COMPREHENSION MONITORING
As observed by Angelo and Cross (1993), "teachers are the closest observers of learning as it takes place in their classrooms-and thus have the opportunity to become the most effective assessors and improvers of their own teaching. But in order for teaching to improve, teachers must first be able to discover when they are off course, how far off they are, and how to get back on the right track." Angelo and Patricia further observe, "The goals of college teachers differ, depending on their disciplines, the specific content of their courses, their students, and their own personal philosophies about the purposes of higher education. All faculty, however, are interested in promoting the cognitive growth and academic skills of their students" (Angelo and Cross, 1993, p. 115). Assessing accomplishments in the cognitive domain has occupied educational psychologists for long, (see, for example, Angelo and Cross (1993), and references therein). Many researchers have worked and developed useful theories and taxonomies on the assessment of academic skills, intellectual development and cognitive abilities, both from the analytical and quantitative point of view. The development of the general theory of measuring the cognitive abilities began with the work of Bloom and others (1956), known as "Bloom Taxonomy." Further developments continued with the contributions of Ausubel (1968), Bloom, Hastings, and Madaus (1971), McKeachie, Pintrich, Lin, and Smith (1986), and Angelo and Cross (1993), among others. "Active engagement in higher learning implies and requires self-awareness and self-direction," which is defined as "metacognition" by cognitive psychologists. For details on metacognition and its applications, see, for example, Brown, Bransford, Ferrara, and Campione (1983), Weinstein and Mayer (1986), and Angelo and Cross (1993), among others.

No matter, what our topic design, classroom strategies, assessment practices and interactions with students may be, it is expected that a teacher uphold the following principles for effective teaching and learning in all classes (from "Education and Research Policy (2000)", Flinders University of South Australia; http://www.flinders.edu.au/teach/teach/home.html). Teaching should:

- focus on desired learning outcomes for students, in the form of knowledge, understanding, skill and attitudes;
- assist students in forming broad conceptual understandings while gaining depth of knowledge;
- encourage informed and critical questioning of accepted theories and views;
- develop an awareness of the limited and provisional nature of much of current knowledge in all fields;
- see how understanding evolves and is subject to challenge and revision;
- engage students as active participants in the learning process, while acknowledging that all learning must involve a complex interplay of active and receptive processes;
- engage students in discussion of ways in which study tasks can be undertaken;
- respect students' right to express views and opinions;
- incorporate a concern for the welfare and progress of individual students;
- proceed from an understanding of students knowledge, capabilities and backgrounds;
encompass a range of perspectives from groups of different ethnic background, socio-economic status and sex;
acknowledge and attempt to meet the demands of students with disabilities;
encourage an awareness of the ethical dimensions of problems and issues;
utilize instructional strategies and tools to enable many different styles of learning and;
adopt assessment methods and tasks appropriate to the desired learning outcomes of the course and topic and to the capabilities of the student.

It is evident, as noted above, that the classroom assessment technique is one of the most significant and important components of classroom research and teaching strategies. There are various classroom assessment techniques developed by Angelo and Cross (1993) which lead to better learning and more effective teaching. The following are some of the objectives of the Classroom Assessment Techniques (CAT’s):

• these CAT’s assess how well students are learning the content of the particular subject or topic they are studying.
• these are designed to give teachers information that will help them improve their course materials and assignments.
• these CAT’s require students to think more carefully about the course work and its relationship to their learning.

Thus, it is clear that the Classroom Assessment Techniques (CAT’s) are designed to help teachers measure the effectiveness of their teaching by finding out what students are learning in the classroom and how well they are learning it. For a detailed analysis of these CAT’s as well as their philosophical and procedural background, see, for example, Angelo and Cross (1993), among others. The kind of learning task or stage of learning assessed by these CAT’s is defined by Norman (1980, p. 46) as accretion, the “accumulation of knowledge into already established structures,” see, for example, Norman (1980), among others, for details. According to Greive (2003, p. 48), “classroom assessment is an ongoing sophisticated feedback mechanism that carries with it specific implications in terms of learning and teaching.” Grieve further observes, “The classroom assessment techniques emphasize the principles of active learning as well as student-centered learning.”

This paper deals with the implementation of three types of Classroom Assessment Techniques, namely, “Course-Related Self-Confidence Surveys,” “Muddiest Point,” and “Exam Evaluations,” in a Business Calculus Class. These techniques are used for assessing:

i. Course-Related Knowledge and Skills;
ii. Learner Attitudes, Values, and Self-Awareness; and
iii. Learner Reactions to Instruction.

The organization of this paper is as follows. Section 2 contains the methods of description, purpose and related teaching goals of using the Classroom Assessment Technique of Course-Related Self-Confidence Surveys (CATORCS), the Muddiest Point (CATMP), and the Exam Evaluations (CATEE). In Section 3, the implementations of these CAT’s in a Business Calculus Class are provided. Section 4 contains the data analysis and discussions of these techniques. Some concluding remarks are presented in Section 5.
2. Methods

This section discusses the description, purpose and related teaching goals of three CAT’s, as stated above.

2.1 The Course-Related Self-Confidence Surveys

2.1.1 Description

The “Course-Related Self-Confidence Surveys (CATCRSCS)” is one of the five Classroom Assessment Techniques (CAT’s) discussed in Angelo and Cross (1993, Chapter 8, p. 255), for assessing learner attitudes, values, and self-awareness, known as “meta-cognition.” It is one of the simplest CAT’s. It provides an efficient avenue of input and a high information return to the instructor without spending much time and energy. It is designed to help teachers better understand and more effectively promote the development of attitudes, opinions, values, and self-awareness that takes place while students are taking their courses. The Course-Related Self-Confidence Surveys help teachers in assessing the students’ level of confidence in their ability to learn the relevant skills and materials. According to Angelo and Cross (1993, pp. 275 - 276), the Classroom Assessment Technique of “Course-Related Self-Confidence Surveys” is useful in the following situations:

a) In courses where students are trying to learn new and unfamiliar skills, or familiar skills that they failed in previous attempts;

b) In introductory courses, such as, in mathematics, public speaking, and natural sciences, before the skills in question are introduced, and again when students are likely to have made significant progress toward mastering them.

2.1.2 Purpose

The following are the main purpose of the Classroom Assessment Technique of “Course-Related Self-Confidence Surveys,” (see, for example, Angelo and Cross, 1993, pp. 275 & 277, for details):

(i) It helps teachers in assessing the students’ level of confidence in their ability to learn the relevant skills and materials;

(ii) It provides information on students’ self-confidence – and, indirectly, on their anxieties – about specific and often controllable elements of the course;

(iii) It helps students learn that a minimum level of confidence is necessary to learning;

(iv) The instructor uses this feedback to guide their teaching strategies to make a particular lesson or topic more clear, lucid, understandable and free from any anxieties.

2.1.3 Related Teaching Goals

The following are related teaching goals of using the “Course-Related Self-Confidence Surveys,” known as Teaching Goal Inventory (TGI), (see, for example,
the Teaching Goal Inventory (TGI), Exhibits 2.1. and 2.2., Angelo and Cross, 1993, pp. 20 – 23, for details):

a) Develop a lifelong love of learning;
b) Develop (self-) management skills;
c) Develop leadership skills;
d) Develop a commitment to personal achievement;
e) Improve self-esteem/self-confidence;
f) Develop a commitment to one’s own values;
g) Cultivate emotional health and well-being;
h) Cultivate physical health and well-being.

2.2 The Muddiest Point

2.2.1 Description

The muddiest point assessment technique is another simplest CAT for assessing course-related knowledge and skills of students, known as “declarative learning,” (see, for example, Angelo and Cross 1993, Chapter 7, p. 115, for details). It provides an efficient avenue of input and a high information return to the instructor without spending much time and energy. In the muddiest point assessment technique, the students are to respond to a single question: “What was the muddiest point in _________?” The students are asked to identify “what they do not understand either about the topic or in the lecture or class.” The focus of the muddiest point assessment technique might be a lecture, a topic, a discussion, a homework assignment, a demonstration, a film, a play, or a general problem-solving activity. Angelo and Cross (1993, p. 155) suggest using the muddiest point assessment technique in the following situations:

a) Quite frequently in classes where a large amount of new information is presented each session – such as mathematics, statistics, economics, health sciences, and the natural sciences – probably because there is a steady stream of possible “muddy points;”
b) In courses where the emphasis is on integrating, synthesizing, and evaluating information.

2.2.2 Purpose

The following are the main purpose of the muddiest point assessment technique:

(i) It provides information on what students find least clear about a particular lesson or topic;
(ii) It provides information on what students find most confusing about a particular lesson or topic;
(iii) The learners quickly identifies what they do not understand and articulate those muddy points;
(iv) The instructor uses this feedback to guide their teaching strategies to make a particular lesson or topic clearer, more lucid, understandable and free from any muddiest point.
2.2.3 Related Teaching Goals

The following are related teaching goals of using the Assessment Technique of “Muddiest Point,” (see, for example, the Teaching Goal Inventory (TGI), the Teaching Goal Inventory (TGI), Exhibits 2.1. and 2.2., Angelo and Cross, 1993, pp. 20 – 23, for details):

(i) Improve skill at paying attention;
(ii) Develop ability to concentrate;
(iii) Improve listening skills;
(iv) Develop appropriate study skills, strategies, and habits;
(v) Learn terms and facts of this subject;
(vi) Learn concepts and theories in this subject.

2.3 The Exam Evaluations

2.3.1 Description

There are various classroom assessment techniques developed by Angelo and Cross (1993) which are directly concerned with better learning, more effective teaching, and assessing learner reactions to instruction. The purpose of this project is also to apply one of the Classroom Assessment Techniques (CAT’s) designed for “Assessing Learner Reactions to Instruction.” These are classified into the following categories: (a) Assessing Learner Reactions to Teachers and Teaching; and (b) Assessing Learner Reactions to Class Activities, Assignments, and Materials, (see, for example, Angelo and Cross, 1993, Chapter 9, p. 317, among others, for details). Each of these categories has five classroom assessment techniques. According to Angelo and Cross (1993), “The second category of these CAT’s is designed to give teachers information that will help them improve their course materials and assignments. At the same time, these CAT’s require students to think more carefully about the course work and its relationship to their learning.” The “Exam Evaluations (CATEE)” is one of the simplest CAT’s, which belongs to this category. It is applicable to many classroom situations. It provides an efficient avenue of input and a high information return to the instructor without spending much time and energy. It is designed to help the instructor to examine both “what the students think that they are learning from exams, tests, or quizzes” and “their evaluations of the fairness, appropriateness, usefulness, and quality of exams, tests, or quizzes.” According to Davis (1999), “Exams, tests, or quizzes are powerful educational tools that serve at least four functions as follows: (I) These exams, tests, or quizzes help the instructors evaluate students and assess whether they are learning what the instructors are expecting them to learn. (II) Well-designed exams, tests, or quizzes serve to motivate and help students structure their academic efforts. The students study in ways that reflect how they think they will be tested. If they expect an exam focused on facts, they will memorize details; if they expect a test that will require problem solving or integrating knowledge, they will work toward understanding and applying information (see, for example, Crooks (1988), McKeachie (1986), and Wergin (1988), among others). (III) The exams, tests, or quizzes can help the instructors understand how successfully the instructors are presenting the material. (IV) Finally, the exams, tests, or quizzes can reinforce learning by providing students with indicators of what topics or skills they have not yet mastered and should concentrate on.” Davis (1999) further observes, “An examination is the most comprehensive
form of testing, typically given at the end of the term (as a final) and one or two times during the semester (as midterms). A test is more limited in scope, focusing on particular aspects of the course material. A course might have three or four tests. A quiz is even more limited and usually is administered in fifteen minutes or less.” For details on exams, tests, and quizzes, general strategies, types, etc., see, for example, Davis (1999), among others, and references therein.

Thus, it is clear from the above that the Classroom Assessment Technique of “Exam Evaluations” helps teachers in assessing the students’ level of confidence in their ability to learn the relevant skills and materials. It is designed to give teachers information that helps them improve their course materials and assignments. At the same time, this CAT requires students to think more carefully about the course work and its relationship to their learning. According to Angelo and Cross (1993, p. 359), the Classroom Assessment Technique of “Exam Evaluations” is useful in the following situations:

- It can be profitably used to get feedback on any substantial quiz, test, or exam.
- To ensure that the memory of quiz, test, or exam is still fresh in students’ minds, the “Exam Evaluation” may be included within the exam itself, as the final section.
- The “Exam Evaluation Form” may be handed out to the students for completion soon after they have finished the exam.

2.3.2 Purpose

The following are the main purpose of the Classroom Assessment Technique of “Exam Evaluations” (see, for example, Angelo and Cross, 1993, p. 359, for details).

- It helps teachers to examine both “what the students think that they are learning from exams, tests, or quizzes” and “their evaluations of the fairness, appropriateness, usefulness, and quality of exams, tests, or quizzes.”
- It provides teachers with specific student reactions to tests and exams, so that they can make the exams more effective as learning and assessment devices.
- It helps teachers in assessing the students’ level of confidence in their ability to learn the relevant skills and materials.
- It provides information on students’ self-confidence – and, indirectly, on their anxieties – about specific and often controllable elements of the course.
- It helps students learn that a certain level of confidence is necessary to learning.
- The instructor uses this feedback to guide their teaching strategies to make a particular lesson or topic more clear, lucid, understandable and free from any anxieties.

2.3.3 Related Teaching Goals

The following are related teaching goals of using the “Exam Evaluations” Technique (see, for example, the Teaching Goal Inventory (TGI), Exhibits 2.1. and 2.2., Angelo and Cross, 1993, pp. 20 – 23 and p. 359, for details).

(i) Develop appropriate study skills, strategies, and habits;
(ii) Learn to evaluate methods and materials in this subject;
(iii) Cultivate an active commitment to honesty;
(iv) Develop capacity to think for oneself.

3. Implementation

This section discusses the implementation of three CAT’s, as described above, in a Business Calculus Class.

3.1 The Course-Related Self-Confidence Surveys

This section discusses the development and implementation of the Classroom Assessment Technique of “Course-Related Self-Confidence Surveys (CATCRSCS)” in a Business Calculus Class. The following topics were already introduced, taught, and discussed in prior lectures of the class before the Course-Related Self-Confidence Surveys were conducted: “Limits and Continuity Concepts.” The prescribed textbook for this Course was: “Calculus for Business, Economics, and the Social and Life Sciences,” 8th edition, by Laurence D. Hoffman and Gerald L. Bradley, McGraw-Hill, 2004, ISBN: 0 - 07-242432 – X.

Calculus is one of the most important and powerful branches of mathematics with a wide range of applications, including curve sketching, optimization of functions, analysis of rates of change, and computation of area and probability. The concepts of limits and continuity form the basis of any rigorous development of the laws and procedures of calculus. In any study of calculus, the concepts of limits and continuity of a function are fundamental. They are primary tools of calculus, and lie at the heart of much of modern mathematics. The limit process involves examining the behavior of a function \( f(x) \) as \( x \) approaches a number \( c \) that may or may not be in the domain of \( f(x) \). On the other hand, a continuous function is one whose graph can be drawn continuously without any break or interruption. There are many practical situations and physical phenomena in which limiting and continuous behavior occurs. The limits and continuity of a function \( f(x) \) are defined as follows:

**DEFINITION 1: LIMIT OF A FUNCTION**

Let \( y = f(x) \) be a function of \( x \). Then a number \( L \) is called the limit of the function \( y = f(x) \) if \( f(x) \) gets closer and closer to \( L \) as \( x \) approaches a number \( c \) that may or may not be in the domain of \( f(x) \). This behavior of the function \( f(x) \) is expressed by writing \( \lim_{x \to c} f(x) = L \).

**DEFINITION 2: EXISTENCE OF LIMIT OF A FUNCTION**

The limit of a function \( f(x) \) at \( x = c \), i.e., \( \lim_{x \to c} f(x) \) exists if and only if the left-hand limit \( \lim_{x \to c^-} f(x) \) and the right-hand limit \( \lim_{x \to c^+} f(x) \) exist and are equal.

**DEFINITION 3: CONTINUITY OF A FUNCTION AT A POINT**
A function \( f(x) \) is said to be continuous at a point \( x = c \) if the following conditions are satisfied:

(i) \( f(c) \) is defined;
(ii) \( \lim_{x \to c} f(x) \) exists;
(iii) \( \lim_{x \to c} f(x) = f(c) \).

The following ideas on limits and continuity of a function \( f(x) \), with illustration by some examples and applications were introduced, defined, and discussed in the class before the surveys, (see, for example, Hoffman and Bradley, 2004, pp. 57 – 79, for details).

- Limit of a function
- Limits at infinity
- Limits at infinity of a rational function
- Infinite limit
- One-sided limits
- Existence of a limit
- Continuity of a function at a point
- Discontinuity of a function at a point
- Limits and Continuity of polynomials, rational and piece-wise defined functions

After introducing and discussing the concepts of limits and continuity of a function \( f(x) \) and illustrating with some examples and applications, the following Course-Related Self-Confidence Surveys (Table 3.1.1) were conducted.

Table 3.1.1

"The Course-Related Self-Confidence Surveys"
(On the Self-Confidence in Limits and Continuity Concepts)
(Students’ Response)

This survey is to help both of us understand your level of confidence in your limit and continuity skills. Rather than thinking about your self-confidence in limits and continuity concepts in general terms, please indicate how confident you feel about your ability to do the various kinds of problems on “Limits and Continuity Concepts” listed below in the Table. (Circle the most accurate response for each.)

<table>
<thead>
<tr>
<th>Items</th>
<th>Kinds of Problems and Concepts</th>
<th>Self-Confidence in Your Ability to Do Them (Students’ Response)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>None Low Medium High Totals</td>
</tr>
<tr>
<td>1</td>
<td>Limit of a function</td>
<td>0 1 2 8 11</td>
</tr>
<tr>
<td>2</td>
<td>Limits at infinity</td>
<td>0 0 9 2 11</td>
</tr>
</tbody>
</table>
The students responded to the question very enthusiastically. Out of 14 students in the class, 11 were present on the day, when the Surveys were conducted. The students’ response (namely, none, low, medium, and high) on the nine components of the concepts of limits and continuity of a function $f(x)$, as discussed in the class, are tabulated in Table 3.1.1 above.

### 3.2 The Muddiest Point

This section discusses the development and implementation of the Classroom Assessment Technique of “Muddiest Point (CATMP),” in the said Business Calculus Class. The “Concepts of the Derivative of a Function” were already introduced, taught, and discussed in the previous lectures of the class before the Muddiest Point (CATMP) Surveys were conducted.

The derivative of a function is a very important concept in calculus and mathematics, in general. It is one of the primary tools for studying the rates of change of a variable with respect to another variable. It is also used to compute the slope of the graph of a function of a variable. Many physical phenomena can also be described through the derivative of a function. It is defined as follows (see, for example, Hoffman and Bradley, 2004, pp. 96 – 104, for details).

1. **Definition:** The derivative of the function $y = f(x)$ with respect to $x$ is the function $f'(x)$ given by

   $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad (1.1)$$

   (Read as “$f$ prime of $x$”). The process of computing the derivative is called differentiation, and $f(x)$ is said to be differentiable at a point $x = c$ if $f'(c)$ exists, i.e., if the above limit (1.1) that defines $f'(x)$ exists when $x = c$.

2. **Notation:** The derivative of $y = f(x)$ is denoted as: $f'(x), \frac{df}{dx}$ or $\frac{dy}{dx}$. 

<table>
<thead>
<tr>
<th></th>
<th>Limits at infinity of a rational function</th>
<th>0</th>
<th>2</th>
<th>7</th>
<th>2</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Infinite limit</td>
<td>0</td>
<td>4</td>
<td>6</td>
<td>1</td>
<td>11</td>
</tr>
<tr>
<td>5</td>
<td>One-sided limits</td>
<td>0</td>
<td>1</td>
<td>10</td>
<td>0</td>
<td>11</td>
</tr>
<tr>
<td>6</td>
<td>Existence of a limit</td>
<td>0</td>
<td>2</td>
<td>9</td>
<td>0</td>
<td>11</td>
</tr>
<tr>
<td>7</td>
<td>Continuity of a function at a point</td>
<td>0</td>
<td>3</td>
<td>6</td>
<td>2</td>
<td>11</td>
</tr>
<tr>
<td>8</td>
<td>Discontinuity of a function at a point</td>
<td>0</td>
<td>3</td>
<td>7</td>
<td>1</td>
<td>11</td>
</tr>
<tr>
<td>9</td>
<td>Limits and Continuity of polynomials, rational and piece-wise defined functions</td>
<td>0</td>
<td>2</td>
<td>9</td>
<td>0</td>
<td>11</td>
</tr>
<tr>
<td>Totals</td>
<td></td>
<td>0</td>
<td>18</td>
<td>65</td>
<td>16</td>
<td>99</td>
</tr>
</tbody>
</table>
3. Slope \((m)\) of the tangent line to the graph of \(y = f(x)\) at a point \((x_0, y_0)\), where \(y_0 = f(x_0)\), is given by the derivative of the function \(y = f(x)\) at \(x_0\), i.e., by \(m = f'(x_0)\).

4. Equation the tangent line to the graph of \(y = f(x)\) at \((x_0, y_0)\) is given by \(y - y_0 = m(x - x_0)\).

After introducing and discussing the concepts of the derivative of a function and illustrating with some examples and applications, the following question was posed during the last ten minutes of the lecture. The students were provided with index cards to answer the question.

> Question: "What was the muddiest point in the concept of the derivative of a function?" The students were asked to identify "what they did not understand about the topic or in the lecture or class." What was the least clear and most confusing point about the topic?

The students responded to the question very enthusiastically. Out of 14 students in the class, 12 were present on that day. Based on the students’ response on the five components of the concept of the derivative of a function as discussed in the class, the muddiest points, namely, most confusing, least clear, and somewhat clear, are given in Table 3.2.1 below. The data analysis is also provided.

**Table 3.2.1**

The Muddiest Point
(on the concept of the derivative of a function)
(Students’ Response)

<table>
<thead>
<tr>
<th>Items</th>
<th>Kinds of Problems</th>
<th>Most Confusing</th>
<th>Least Clear</th>
<th>Somewhat Clear</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Definition: The derivative of the function (y = f(x)) with respect to (x)</td>
<td>3</td>
<td>0</td>
<td>9</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>2. Notation: (\frac{dy}{dx})</td>
<td>3</td>
<td>1</td>
<td>8</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>3. Slope ((m)) of the tangent line to the graph of (y = f(x)) at ((x_0, y_0)).</td>
<td>3</td>
<td>2</td>
<td>7</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>4. Equation the tangent line to the graph of (y = f(x)) at ((x_0, y_0)).</td>
<td>4</td>
<td>1</td>
<td>7</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>5. Applications (Examples)</td>
<td>4</td>
<td>2</td>
<td>6</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>Totals</td>
<td>17</td>
<td>6</td>
<td>37</td>
<td>60</td>
<td></td>
</tr>
</tbody>
</table>
3.3 The Exam Evaluations

This section discusses the development and implementation of the Classroom Assessment Technique of “Exam Evaluations (CATEE)” to one of the tests, i.e., Test # 1, in the said Business Calculus Class. Test # 1 was already administered in the class (the details of which are provided in Appendix I). After administering Test # 1, the following Exam Evaluations Surveys were conducted (see Table 3.3.1).

Table 3.3.1
Sample Survey: Exam Evaluations (of Test # 1)

<table>
<thead>
<tr>
<th>Questions</th>
<th>Exam Evaluations (of Test # 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Did you feel that the test was fairer assessment of your learning of the materials covered before the test?</td>
<td>Fair</td>
</tr>
<tr>
<td>2. Did you enjoy the content or form of the test?</td>
<td>Content</td>
</tr>
<tr>
<td>3. Did you learn more from free-response questions (by solving the given problems) than from the multiple-choice questions?</td>
<td>From free-response</td>
</tr>
<tr>
<td>4. What type of test would you prefer as your remaining tests and final exam during the rest of the semester?</td>
<td>Free-response questions (by solving the given problems)</td>
</tr>
</tbody>
</table>

The students responded to the question very enthusiastically. Out of 14 students in the class, 13 were present on that day. The data analysis of students’ responses on each four components of Test # 1 (see Table 3.3.1 above) is discussed below.
4. Data Analysis and Discussions

This section discusses the data analysis of the implementation of three CAT’s, as described above, in the said Business Calculus Class, i.e., CATCRSCS, CATMP, and CATEE.

4.1 CATCRSCS

Using MINITAB, the following bar graph, (see Figure 4.1.1 below), was drawn based on the students’ response on nine components of the concepts of limits and continuity of a function $f(x)$. During the survey, 11 out of 14 students were present in the class on that day. The total number of response was 99. It is clear that most of the students responded with “Medium” (i.e. 65.66 %) on all nine components. Approximately 18.18 % of the students’ response was “Low,” whereas 16.16 % students’ response was “High.” No student responded with “None” on the nine components. It is also clear from Table 3.1 that 72 % of Students’ Response on “Self-Confidence in Your Ability to Do Them” for the concept “Limit of a function” was “High.” 82 % of the students’ response was “Medium” for each of “Limits at infinity,” “Existence of a limit,” and “Limits and Continuity of polynomials, rational and piece-wise defined functions,” whereas 91 % was “Medium” for “One-sided limits.”

![Figure 4.1.1](image-url)

“COURSE-RELATED SELF-CONFIDENCE SURVEYS”
(On the Self-Confidence in Limits and Continuity Concepts)
(Students’ Response)

Percent within all data.
4.2 CATMP

Using MINITAB, the following bar graphs were drawn based on the students’ response on five components of the concept of the derivative of a function as discussed in the class (see Table 3.2.1 above). These are provided in the Figure 4.2.1 below. It is clear that most of the students responded as “Somewhat Clear” (i.e. 61.65 %) on all five components. Approximately 28.33 % of the students’ response was “Most Confusing,” whereas 10 % students’ response was “Least Clear.”

![Figure 4.2.1](image_url)

**Percent within levels of Class.**

4.3 CATEE

Using MINITAB and PHStat, the following graphs, (see Figures 4.3.1, and 4.3.2 below), were drawn based on the students’ response. During the survey, 13 out of 14 students were present in the class on that day. The total number of response was 52. As per analysis of the students’ response to survey questions, we observed:

(i) That 14 % was “Both Content and Form” for survey question # 2;

(ii) That 15 % was “Form Both” for survey question # 3;

(iii) That 15 % was “Both Free-Response and Multiple-Choice Questions” for survey question # 4;

(iv) That Approximately 10 % of the students’ response was “Fair” and “All of these” for survey questions # 1.
For responses to other questions, see Figures 4.3.1 and 4.3.2 below.

Figure 4.3.1

Classroom Assessment Technique - Sample Survey: Exam Evaluations (Test # 1)
From the above analysis of data, it is easily observed:

(A) That most of the students of the said Business Calculus Class had the same response during the “COURSE-RELATED SELF-CONFIDENCE SURVEYS on the Self-Confidence in Limits and Continuity Concepts,” i.e., most of the students responded with “Medium” (i.e. 65.66 %) on all nine components.

(B) That most of the students mentioned the same “muddy point”: the concept of the derivative of a function is “Somewhat Clear,” but, at the same time, it is either “Most Confusing” or “Least Clear” to them.

(C) That the students’ responses were very encouraging, as most of them enjoyed Test # 1. They were able to apply the already-taught concepts to answer both free-response questions and the multiple-choice questions. Most of the students had the following opinion about Test # 1.

- They felt that the test was a fair assessment of their learning of the materials covered before the test.
- They enjoyed both the content and form of the test.
- They felt that they learnt more from both free-response questions (by solving the given problems) and the multiple-choice questions.
- Their preference was both free-response questions (by solving the given problems) and the multiple-choice questions for the remaining tests and final exam during the rest of the semester.
7. Concluding Remarks

Based on our observations and analysis, it is clear that the three CAT’s considered in this project, i.e., the Course-Related Self-Confidence Surveys (CATCRSCS), the Muddiest Point (CATMP), and the Exam Evaluations (CATEE), are the simplest and most important Classroom Assessment Techniques. These Classroom Assessment Techniques help teachers to measure the effectiveness of their teaching by finding out what students are learning in the classroom and how well they are learning. In addition, these techniques provide an efficient avenue of input and a high information return to the instructor without spending much time and energy. It is recommended that, in future, more techniques be developed and implemented in other mathematics classes, for example, college preparatory mathematics, college level mathematics, etc., for better learning and more effective teaching.

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References


MCKEACHIE, W. J., PINTRICH, P. R., LIN, YI-GUANG, and SMITH, D. A. F. (1986), *Teaching and Learning in the College Classroom: A Review of the Research Literature, National Center for Research to Improve Postsecondary Teaching and Learning, University of Michigan, Ann Arbor*.


### Appendix I

NAME: ______________________________      Student ID: __________________

MAC 2233: CALCULUS FOR BUSINESS

Test # 1

DIRECTIONS: Answer ALL questions. Total Points: 100.

PART A (80 Points)
(Show your work for full credit.)

(1) Find the limit:
\[
\lim_{{x \to 2}} \frac{x - 2}{x^2 - 4}
\]

(2) Find the limit:
\[
\lim_{{x \to \infty}} \frac{x^2 + 3x + 2}{x^2 - 1}
\]
(3) Differentiate the following function:

\[ f(x) = \frac{1}{3}x^7 - 2x^5 + 9x - 8 \]

<table>
<thead>
<tr>
<th>(4) Differentiate the following function:</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ f(x) = \frac{x^2}{x - 2} ]</td>
</tr>
</tbody>
</table>

(5) Test the continuity of the following function at \( x = 3 \): 

\[ f(x) = \begin{cases} 
  x^2 & \text{if } x \leq 3 \\
  9 & \text{if } x > 3 
\end{cases} \]

by showing the following steps:

(a) Find \( f(3) = \)

(b) Find the following limits for the above function:

(i) Right-hand limit: \( \lim_{x \to 3^+} f(x) = \)

(ii) Left-hand limit: \( \lim_{x \to 3^-} f(x) = \)

(iii) Does \( \lim_{x \to 3} f(x) \) exist? If it exists, what is \( \lim_{x \to 3} f(x) = \)

(c) Is \( f(x) \) continuous at \( x = 3 \)? State the reason(s).

(6) If \( f(x) = x^2 - 1 \), using the definition of derivative, find the first derivative of the function, as given below.

\[ f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \]

Hence find the slope and equation of the line that is tangent to the graph of the given function at \( x = -1 \).

**PART B (20 Points)**

Multiple-Choice Questions
(Circle your answers)

(7) Differentiate

\[ f(x) = (x^2 - 1)(x - 3) \]

(a) \( 3x^2 - 6x - 1 \)  
(b) \( 6x + 1 \)  
(c) \( x^2 + 1 \)  
(d) \( x^2 + 3x^2 + 6x + 1 \)

(8) True or false: The left-hand limit of the function given below, i.e., 

\[ f(x) = \begin{cases} 
  x^2 & \text{if } x \leq 2 \\
  x + 2 & \text{if } x > 2 
\end{cases} \]

(a) True  
(b) False
(9) True or false: The right-hand limit of the function given below, i.e., \( \lim_{x \to 3^+} f(x) \), where
\[
f(x) = \begin{cases} 
    x & \text{if } x < 3 \\
    x + 1 & \text{if } x \geq 3
\end{cases}
\]
is \( 3 \).
(a) True  (b) False

(10) The derivative of
\[
f(x) = \frac{2}{\sqrt{x}}
\]
is
(a) \(-\frac{1}{x^{3/2}}\)
(b) \(\frac{1}{x^{3/2}}\)
(c) \(-\frac{1}{\sqrt{x}}\)
(d) \(-\sqrt{x}\)