SUMMARY: “SETS and LOGIC”

SETS

Notation:

1. listing its elements in a “Roster”
   for example: A={a,e,i,o,u}. NOTE: when the list is extensive and the pattern is obvious, an ellipsis may be used; for example: B={a,b,c,. . . ,z}

2. giving a description of the elements that belong in the set
   for example: A={all vowels in English alphabet} and B={all letters in the English alphabet}

3. using set builder notation
   for example: A={x| x > 0}

Special sets:

Universal set: The set of all elements under discussion. The notation for the universal set is set U.

Subset: Set A is a subset of Set B if Set A contains no element that is not also an element of Set B. The notation is A \subseteq B.

Null or empty set: A set containing no elements. Symbols are \emptyset and { }.

Number of subsets: If a set contains “n” elements then the number of subsets is equal to 2^n
   NOTE: The empty set is considered a subset of every set and every set is considered a subset of itself.

Set Operations:

Complement: The set of all members of U that are not in A is called the complement of A. The notation is A'.

The intersection of sets A and B is the set of elements that are members of both A and B. The notation is A \cap B.

The union of sets A and B is the set of elements that are in at least one of the sets A or B. That is, all elements that are in A or B or both. The notation is A \cup B.

DeMorgan’s Laws:

(A \cap B)' = A' \cup B'
(A \cup B)' = A' \cap B'

LOGIC

Basic Definitions:

Negation “Not p”: The notation for the negation of p is \neg p
   A statement and its negation always have opposite truth values.

Conjunction “p and q” The notation is p \land q: The conjunction of two statements is true if both statements are true.

Disjunction “p or q” The notation is p \lor q: The disjunction of two statements is true if at least one of the statements is true [that is: one or the other or both are true]
Conditional “If p, then q.” The notation is \( p \rightarrow q \): A conditional statement is false only when the “If-clause” is true and the “Then-clause” is false.

**Truth tables for compound statements:**

<table>
<thead>
<tr>
<th></th>
<th>Negation</th>
<th>Conjunction</th>
<th>Disjunction</th>
<th>Conditional</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>~P</td>
<td>( p \land q )</td>
<td>( p \lor q )</td>
<td>( p \rightarrow q )</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
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<tr>
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<td>T</td>
<td>F</td>
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<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

**Other Forms of the Conditional:**

<table>
<thead>
<tr>
<th></th>
<th>original conditional</th>
<th>( p \rightarrow q )</th>
<th>equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>converse</td>
<td>( q \rightarrow p )</td>
<td>Not equivalent</td>
<td></td>
</tr>
<tr>
<td>inverse</td>
<td>( \sim p \rightarrow \sim q )</td>
<td>Not equivalent</td>
<td></td>
</tr>
<tr>
<td>contrapositive</td>
<td>( \sim q \rightarrow \sim p )</td>
<td>equivalent</td>
<td></td>
</tr>
</tbody>
</table>

**Equivalences and negations of statements:**

<table>
<thead>
<tr>
<th>Statement</th>
<th>Equivalent</th>
<th>Negation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p \rightarrow q ) (If p, then q)</td>
<td>1) ( \sim q \rightarrow \sim p ) (If not q, then not p)</td>
<td>1) ( \sim (p \rightarrow q) ) (It is not true that if p, then q)</td>
</tr>
<tr>
<td></td>
<td>2) ( \sim p \lor q ) (Not p, or q)</td>
<td>2) ( p \land \sim q ) (p AND not q)</td>
</tr>
<tr>
<td>( p \lor q ) (p or q)</td>
<td>1) ( \sim p \rightarrow q ) (If not p, then q)</td>
<td>1) ( \sim (p \lor q) ) (It is not true that p or q)</td>
</tr>
<tr>
<td></td>
<td>2) ( \sim q \rightarrow p ) (If not q, then p)</td>
<td>2) ( \sim p \land \sim q ) (Not p AND not q)</td>
</tr>
<tr>
<td></td>
<td>3) ( q \lor p ) (q or p)</td>
<td>3) ( q \land p ) (q and p)</td>
</tr>
<tr>
<td>( p \land q ) (p and q)</td>
<td>1) ( q \land p ) (q and p)</td>
<td>1) ( \sim (p \land q) ) (It is not true that p and q)</td>
</tr>
</tbody>
</table>
2) \( \sim p \lor \sim q \)  
(not \( p \) OR not \( q \))

**Equivalences and Negations for statements involving Quantifiers**

<table>
<thead>
<tr>
<th>STATEMENT</th>
<th>NEGATION</th>
<th>EQUIVALENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>All A’s are B’s</td>
<td>Some A’s are not B’s</td>
<td>There are no A’s that are not B’s.</td>
</tr>
<tr>
<td>Some A’s are B’s</td>
<td>No A’s are B’s</td>
<td>At least one A is a B.</td>
</tr>
<tr>
<td>Some A’s are not B’s</td>
<td>All A’s are B’s</td>
<td>Not all A’s are B’s.</td>
</tr>
<tr>
<td>No A’s are B’s</td>
<td>Some A’s are B’s</td>
<td>All A’s are not B’s.</td>
</tr>
</tbody>
</table>

**Determining whether an argument is invalid or valid:**

1. If premises contain “all,” “some,” or “no,” use Euler circles:  
   **YOU TRY TO MAKE A DRAWING THAT SHOWS THE PREMISES BUT NOT THE CONCLUSION. IF YOU CAN DO THAT, THEN THE ARGUMENT IS INVALID.**

<table>
<thead>
<tr>
<th>DIAGRAM</th>
<th>ENGLISH</th>
<th>EXAMPLES</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Diagram 1" /></td>
<td>No A’s are B’s</td>
<td>Snakes are not pretty.</td>
</tr>
<tr>
<td><img src="image2.png" alt="Diagram 2" /></td>
<td>No B’s are A’s</td>
<td>No math course is easy.</td>
</tr>
<tr>
<td><img src="image3.png" alt="Diagram 3" /></td>
<td>Some A’s are B’s (at least one A is a B)</td>
<td>Some roses are red.</td>
</tr>
<tr>
<td></td>
<td>Some B’s are A’s (at least one B is an A)</td>
<td>Some rivers are polluted.</td>
</tr>
<tr>
<td></td>
<td>All A’s are B’s</td>
<td>All sharks scare me.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Apples are good for you.</td>
</tr>
</tbody>
</table>

**NOTE:** This is not commutative since only some B’s are A’s.
Marta is a scientist.
Bob swims.

II. If at least one of the premises is an “if . . . then” statement or an “or” statement, use symbolic logic:

In general, an argument is valid if $P_1 \land P_2 \rightarrow C$ is a tautology. In most cases arguments will be recognizable as fitting one of the valid or invalid forms.

<table>
<thead>
<tr>
<th>Summary of Valid Forms of Arguments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type 1</td>
</tr>
<tr>
<td>$p \rightarrow q$</td>
</tr>
<tr>
<td>$\neg p$</td>
</tr>
<tr>
<td>$\therefore q$</td>
</tr>
</tbody>
</table>

Common errors made in drawing conclusions or determining the validity of an argument are:
1. Arguing the converse (assuming in a conditional that if $q$ happens, $p$ must happen).
2. Arguing the inverse (assuming in a conditional that if $p$ didn't happen, $q$ can't happen).
3. Using the exclusive “or” (assuming in a disjunction that if $p$ happens, $q$ can’t happen).

Use of premises that are conjunctions, and conclusions of valid arguments:
If a premise is a conjunction, then both its parts must be true and each can be used separately as a premise.
If an argument has been shown to be VALID then its conclusion must be true, and it can be used as a premise for another argument.

Miscellaneous comments about premises:
1. A premise such as “mathematicians enjoy puzzles” can be read as either:
a) “all mathematicians enjoy puzzles” (Euler circles) or,
b) “if you are a mathematician, then you enjoy puzzles” (symbolic logic)
In other words, “all A are B” is equivalent to “if A then B”.

2. A premise such as “Bob is a mathematician” can be read as either:
a) “Bob is an element of the set of mathematicians” (Euler circles) or,
b) The event of having a mathematician has occurred. (symbolic logic)
SUMMARY OF GEOMETRY

MEASUREMENT SYSTEMS:

METRIC SYSTEM
BASIC units: linear: \textit{m} is a \textit{meter}; weight or mass: \textit{g} is a \textit{gram}
capacity: \textit{l} is a \textit{liter}

PREFIXES:
- \textit{m-} is a \textit{milli-} (key number is 1000);
- \textit{c-} is a \textit{centi-} (key number is 100);
- \textit{k-} is a \textit{kilo-} (key number is 1000);

CONVERSION:

- Using concept that if you are converting units to smaller units, you will need more of the smaller units for the same measurement and if you are converting to larger units you will need fewer of the larger units:
  a) If converting from \textit{x} large units to smaller units: multiply \textit{x} by the key number of the prefix.
  b) If converting from \textit{x} small units to larger units: divide \textit{x} by the key number of the prefix

- Alternate method: Since you are multiplying or dividing by a power of 10, the conversion can be accomplished by moving the decimal point to the right (if multiplying) or to the left (if dividing) the appropriate number of places.

To help arrive at the correct directions and appropriate number of places, you can use the memory aid of:

\begin{center}
\textbf{K H D M D C M} \quad (King Henry died miserably doing college math).
\end{center}

Place the number under the letter representing its unit and note the direction and places to get to the letter of the unit you want to convert to.

For example: Convert 5400 cm to kilometers:

\begin{center}
\begin{tabular}{cccc}
K & H & D & M  \\
M & D & C & M \\
\end{tabular}
\end{center}

\begin{center}
\text{} .054 \quad \leftarrow \quad \leftarrow \quad \leftarrow \\
\text{} 5400. \quad \text{move decimal 5 places to the left.}
\end{center}

ENGLISH SYSTEM

- You must use “conversion fractions” to get from one measurement to another.

Ex:  Convert 72 cups to gallons.
\[
\frac{72 \text{ c}}{1} \cdot \frac{1 \text{ pt}}{2 \text{ c}} \cdot \frac{1 \text{ qt}}{2 \text{ pt}} \cdot \frac{1 \text{ gal}}{4 \text{ qt}} = \frac{9}{2} \text{ gal} = 4.5 \text{ gal}
\]

- When rounding feet and inches to nearest foot, you must have 6 or more inches to round up.
- When rounding pounds and ounces to the nearest pound, you must have 8 ounces or more to round up.
- When rounding a ruler measurement to the nearest fraction of an inch, the final answer may be in any larger unit. For example, given something that measures between 1\(\frac{1}{2}\) in. and 1\(\frac{3}{4}\) in. and asked to find the measure to the nearest quarter inch, the answer could be 1\(\frac{1}{2}\) inches, since 1\(\frac{1}{2}\) inches is the same as 1\(\frac{1}{4}\) inches.

**ANGLES** are measured in degrees using a curved ruler called a protractor:
- acute angle < 90°
- right angle = 90°
- obtuse angle > 90°
- straight angle = 180°

**PAIRS OF ANGLES:**
- **Adjacent angles:** angles that share a common vertex and a common side, but have no other intersection.
  NOTE: in a polygon, adjacent angles share only a common side; they are also referred to as consecutive angles.

- **Complementary angles:** Two angles whose measures have a sum of 90°. If they are adjacent they form a right angle.

- **Supplementary angles:** Two angles whose measures have a sum of 180°. If they are adjacent they form a straight angle.

- **Vertical angles:** Two non-adjacent angles formed by intersecting lines. Vertical angles are equal in measure.

**PARALLEL LINES CROSSED BY ONE TRANSVERSAL**
Two groups of four angles are formed:

Pairs of **corresponding angles** are $\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$, $\angle 4$ and $\angle 8$.

Pairs of **alternate interior angles** are $\angle 4$ and $\angle 6$; $\angle 3$ and $\angle 5$

**NOTE**: If corresponding angles or alternate interior angles are NOT equal, then the lines are NOT parallel.

If the lines are parallel, then

- all acute angles are equal
- all obtuse angles are equal
- any acute angle and any obtuse angle are supplementary

**TRIANGLES**

The sum of the interior angles of a triangle is 180°.

Since all triangles have at least 2 acute angles, a triangle may be classified by the nature of its third angle, that is:

- **an acute triangle** if the third angle is also acute.
- **an obtuse triangle** if the third angle is obtuse.
- **a right triangle** if the third angle is a right angle.

Triangles may be classified by sides:

- **Scalene** -- no sides are equal (and no angles are equal).
- **Isosceles** -- two sides are equal (and the angles opposite the equal sides are equal).
- **Equilateral** -- all three sides are equal (each angle measures 60°; thus, the triangle is also equiangular).

The shortest side of a triangle is opposite the smallest angle; the longest side is opposite the longest side.

**PYTHAGOREAN THEOREM** *(applies to right triangles)*:

The square of the hypotenuse equals the sum of the squares of the other two sides.

$$H^2 = S_1^2 + S_2^2 \quad \text{or} \quad a^2 + b^2 = c^2$$

(where $c$ is the hypotenuse)

**SIMILAR TRIANGLES**:

When two triangles are similar, the measures of corresponding angles are equal, and the lengths of corresponding sides are in proportion. That is, if $\triangle ABC \sim \triangle PQR$, then:

1) $\angle A = \angle P$, $\angle B = \angle Q$ and $\angle C = \angle R$, and
2) \[ \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} \]

Two triangles will be similar if:
- two angles of one triangle are equal to two angles of the other triangle, or
- three pairs of corresponding sides are proportional, or
- two pairs of sides are proportional and the included angles are equal.

If the triangles are similar and corresponding sides are equal, the triangles are congruent.

**POLYGONS:**

<table>
<thead>
<tr>
<th>NUMBER OF SIDES</th>
<th>NAME</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Triangle</td>
</tr>
<tr>
<td>4</td>
<td>Quadrilateral</td>
</tr>
<tr>
<td>5</td>
<td>Pentagon</td>
</tr>
<tr>
<td>6</td>
<td>Hexagon</td>
</tr>
<tr>
<td>8</td>
<td>Octagon</td>
</tr>
</tbody>
</table>

**AN EXTERIOR ANGLE OF A POLYGON** is formed by one side and the extension of the other side at that vertex.
- The exterior angle and the interior angle at any vertex of any polygon are supplementary.
- The exterior angle of a triangle is equal to the sum of the two remote interior angles.

**Summary of Angle Measures for Polygons (in degrees):**

<table>
<thead>
<tr>
<th>Angle Type</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Interior angle</strong></td>
<td>( (n - 2) \times 180 )</td>
</tr>
<tr>
<td><strong>Exterior angle</strong></td>
<td>( \frac{360}{n} )</td>
</tr>
<tr>
<td><strong>Measure of one (regular polygon)</strong></td>
<td>( 180 - \frac{360}{n} )</td>
</tr>
</tbody>
</table>
QUADRILATERALS

QUADRILATERAL: (4-sided figure)

- no parallel sides: TRAPEZOID
- 1 pair of parallel sides
- 2 pairs of parallel sides: PARALLELOGRAM

NOTE: DIAGONALS:
- of a PARALLELOGRAM bisect each other
- of a RECTANGLE bisect each other and are equal
- of a RHOMBUS bisect each other and are perpendicular
- of a SQUARE bisect each other and are equal and perpendicular

DISTANCE AROUND AN OBJECT:

**PERIMETER** is the linear measure of the distance around a figure.

To find the **perimeter** of a polygon, add the lengths of all of its sides.

**CIRCUMFERENCE** is the linear measure of the distance around a **circle**.

Circumference is Pi times the diameter or Pi times twice the radius.

\[ C = \pi d \quad \text{or} \quad C = 2\pi r \]

MEASUREMENT OF GEOMETRIC FIGURES:

1) **Linear**: used when measuring length, width, depth, distance around an object (in., ft., yd., m., cm, km, etc.)
2) **Square**: used when measuring area (in^2, ft^2, yd^2, m^2, cm^2, km^2, etc.)
3) **Cubic**: used when measuring volume (in^3, ft^3, yd^3, m^3, cm^3, km^3, etc.)
Area Formulas

1. Rectangle  
   \[ A = l \cdot w \]  
   (length x width)

2. Square  
   \[ A = s^2 \]  
   (side squared)

3. Parallelogram  
   \[ A = b \cdot h \]  
   (base x height)

4. Triangle  
   \[ A = \frac{1}{2} b \cdot h \]  
   (1/2 base x height)

5. Trapezoid  
   \[ A = \frac{1}{2} (b_1 + b_2) \cdot h \]  
   (1/2 sum of bases x height)

6. Circle  
   \[ A = \pi r^2 \]  
   (pi x radius squared)
   
   (Circumference:  \( C = \pi d \) or \( 2\pi r \))

Notes:
1. Area measures the amount of surface enclosed in a region and is measured in square units:
   - sq. in. = in\(^2\) or sq. cm = cm\(^2\)
   - NOTE: 9 sq. ft = 1 sq. yd.
2. The area of any figure made up of a combination of the above figures can be found by adding up the individual parts.
3. The area of a shaded region can be found by first finding the figure’s total area and then subtracting out the unshaded area.
4. Leave area and circumference in terms of \( \pi \) unless otherwise indicated.

Volume Formulas

In general, the volume of a solid figure (in cubic units) will be:
   a) the area of the base \cdot height, if the figure goes “straight up” from the base.
   b) \( \frac{1}{3} \) the area of the base \cdot height, if the figure rises to a point (e.g. cone or pyramid).
1. Rectangular Solid (box)  
   Volume = base area \cdot height 
   \[ = \text{lw} \cdot \text{h} \]
   (Surface Area = area of 6 faces = 2\cdot \text{lw} + 2\cdot \text{lh} + 2\cdot \text{wh})

2. Cube  
   \[ \text{Volume} = \text{base area} \cdot \text{height} = \text{e}^2 \cdot \text{e} = \text{e}^3 \]

3. Cylinder  
   \[ \text{Volume} = \text{base area} \cdot \text{height} = \pi \cdot \text{r}^2 \cdot \text{h} = \pi \cdot \text{r}^2 \cdot \text{h} \]
   (Surface Area = area of ends + area of "label")
   \[ = 2 \cdot \pi \cdot \text{r}^2 + 2 \cdot \pi \cdot \text{r} \cdot \text{h} \]

4. Pyramid  
   \[ \text{Volume} = \frac{1}{3} \cdot \text{base area} \cdot \text{height} = \frac{1}{3} \cdot \text{lw} \cdot \text{h} = \frac{1}{3} \cdot \text{lwh} \]

5. Cone  
   \[ \text{Volume} = \frac{1}{3} \cdot \text{base area} \cdot \text{height} = \frac{1}{3} \cdot \pi \cdot \text{r}^2 \cdot \text{h} = \frac{1}{3} \cdot \pi \cdot \text{r}^2 \cdot \text{h} \]

6. Sphere  
   \[ \text{Volume} = \frac{4}{3} \cdot \pi \cdot \text{r}^3 \]
SUMMARY OF PROBABILITY

Probability Of A Single Event Occurring:
- Theoretical Probability: \( P(A) = \frac{n(A)}{n(S)} \), that is the number of outcomes that "match" A over the total number of possible outcomes of the experiment.

Events That Cannot Occur Or Must Occur; Complement Probability:
- If an event cannot possibly occur, then its probability is 0.
- If an event is certain to occur, then its probability is 1.
- If \( P(A) \) represents the probability that event A will occur, then \( P(A') \) represents the probability that A will not occur.

Conditional Probability: \( P(B|A) \) represents the probability of B occurring given that A has occurred. Being given that some event has already occurred can have the impact of making the sample space smaller.

For example, suppose you draw a single card from a standard deck of 52. The probability the card is a king is: \( P(K) = \frac{4}{52} = \frac{1}{13} \).
Probability From Data Given In Table Form

**PROBABILITY OF “AT LEAST ONE…”:**  
None and at least one are complements. So to compute the probability of “at least one”, first find the probability that “none are” and then find its complement. That is:  
Step 1: Compute P(none are….)

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
</tr>
<tr>
<td>B</td>
<td>O</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$P(A$ or $2) = \text{Sum of circles over total of all boxes}$

<table>
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<th></th>
<th>1</th>
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<th>3</th>
<th>4</th>
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<tbody>
<tr>
<td>A</td>
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<tr>
<td>B</td>
<td></td>
<td></td>
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</tbody>
</table>

$P(A) \text{ if given 2} = \text{amount in square over total in circle}$

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
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</tbody>
</table>

$P(2) \text{ if given A} = \text{amount in square over total in circle}$

**ODDS:**  
Whenever a problem involves odds, first make a three-number summary:

$$\frac{F}{T} \quad \text{(# for)}$$

$$\frac{A}{T} \quad \text{(# against)}$$

You can then switch back and forth between odds and probability by using the fractions below:

Odds in favor (for) = $\frac{F}{A}$ or $F:A$  
Odds against = $\frac{A}{F}$ or $A:F$

Probability for = $\frac{F}{T}$  
Prob. against = $\frac{A}{T}$
Counting Rules

**Fundamental Counting Principle**
- is used when a number of distinct tasks are to be done, in which case the number of ways all of these tasks can be accomplished is equal to the product of the number of ways each can be done.
- is used to determine the number of ways \( n \) distinct things can be arranged, that is, when the order matters.

**Combinations** are used to count the number of ways \( n \) things can be taken \( r \) at a time, if the order does not matter.

\[
n C_r = \frac{n!}{r!(n-r)!}
\]

Note: \( n! = n(n-1)(n-2)\ldots(3)(2)(1) \)

**BINOMIAL PROBABILITY (OPTIONAL)**

**Characteristics of a Binomial Experiment:**
- \( n \) repeated independent trials
- Each trial has just two possible outcomes, success (\( S \)) and failure (\( F \)).
- If \( p = P(S) \) and \( q = P(F) \), then \( p + q = 1 \)

To find the probability of an exact number \( (x) \) of Successes, find the probability of all the successes occurring first followed by the failures, and then multiply by the number of ways the \( n \) trials could be taken \( x \) at a time.

The formula for computing this is: 
\[
P(x \text{ successes}) = (nC_x)p^xq^{n-x}
\]
SUMMARY OF STATISTICS

REQUIREMENTS FOR AN **UNBIASED SAMPLE**
1. The sample must be selected only from the target population.
2. Each member of the population must have an equal chance of being in the sample.
3. Some type of random selection method must be used.

**GRAPHS:**
Line, Bar, Circle, Scatter diagrams

NOTE: Trends and relations can be inferred but *cause and effect cannot* be determined by graphs alone.

Using circle graphs two types of questions can be asked:
I. Given percentages and total amount; II. Given numbers, find percent of total find actual number

A circle represents the whole therefore
A circle represents the whole therefore
the percents add to 100% add the numbers to find the “total”

[Multiplier % (in decimals) by total amount] [Use ratio of part to total; change into %]

**MEASURES OF CENTRAL TENDENCY**
The mean (average) is the sum of the data divided by the number of pieces of data. It is sensitive to extreme values.
The median is the value in the middle of a set of ranked data. It is equivalent to the 50th percentile.

To find the median:
1. Be sure the scores (data) are in ranked order (“size places”)
2. If there are an odd number of scores, the median is the middle score. For instance, if there are 19 scores, half of 19 is 9½. So the 10th score would be the median.
3. If there are an even number, use the average of the two scores in the middle. For instance, if there are 50 scores, then there are 25 in each half. The 25th score is the last in the first half and the 26th is the first one of
the second half. The median would be the average of the 25th and 26th scores.

The mode is the data item that occurs most frequently. If no data item occurs more frequently than any other, there is said to be no mode.

**IN NORMALLY DISTRIBUTED DATA**, the mean, median, and mode are equal.

**SKewed DATA**: If data contains rare, extreme values to the right on the scale the mean will lie to the right of the mode and the median. The curve is said to be **skewed to the right** (or skewed positively). If the rare, extreme values are to the left, the curve will be **skewed to the left** (or skewed negatively).

![Skewed right and left diagrams]

**FREQUENCY TABLES** When there are many occurrences of some different scores they are usually presented in a table that lists the different scores in one column and gives the frequency of occurrence (how often they occur) in another column.

<table>
<thead>
<tr>
<th>Score</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>3</td>
</tr>
<tr>
<td>70</td>
<td>5</td>
</tr>
<tr>
<td>60</td>
<td>8</td>
</tr>
<tr>
<td>50</td>
<td>4</td>
</tr>
</tbody>
</table>

**Finding the modal number (mode)**: There are four 50’s, eight 60’s, five 70’s and three 80’s. Since there are more 60’s than any other score, the mode is 60.

<table>
<thead>
<tr>
<th>Score</th>
<th>Frequency</th>
<th>Contains scores</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>3</td>
<td>$x_1, x_2, x_3$</td>
</tr>
<tr>
<td>70</td>
<td>5</td>
<td>$x_4, x_5, x_6, x_7, x_8$</td>
</tr>
</tbody>
</table>

**Finding the median**: There are 20 scores, so there are 10 in the upper half and 10 in the lower half. The average of the 10th
and $11^{th}$ score is the median. Since both the $10^{th}$ & $11^{th}$ scores are 60, the median is 60.

<table>
<thead>
<tr>
<th>Score</th>
<th>Frequency</th>
<th>Score x frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>3</td>
<td>$3 \times 80 = 240$</td>
</tr>
<tr>
<td>70</td>
<td>5</td>
<td>$5 \times 70 = 350$</td>
</tr>
<tr>
<td>60</td>
<td>8</td>
<td>$8 \times 60 = 480$</td>
</tr>
<tr>
<td>50</td>
<td>4</td>
<td>$4 \times 50 = 200$</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Total = 1270</strong></td>
</tr>
</tbody>
</table>

Finding the mean: We multiply each score by its frequency, add up the products and then divide by the total number of scores (namely 20).  

$\frac{1270}{20} = 63.5$  

So the mean is 63.5.

**PERCENTILE RANK** indicates the percent of the ranked data that lies below the given score.

I. a. If asked for the percent of data below a given data item (score, measurement, etc) look for that score in the left column, and then read the percentile directly from the other column.

b. If asked to find the data item (score, measurement, etc) such that a given percent of the data is less (or lower) look in the percentile column for the given percent and read the data item directly from the other column.

II. If the question deals with information above a percent or score subtract the given or required percent from 100% and then follow the method outlined in Ia or Ib.

III. If asked for the percent of data between two given data items (scores, measurements, etc) or percentiles subtract smaller from larger.

**Examples:**

What % scored below 66? 58% of the scores were below what score?
What % scored above 66?  
42% of the scores were above what score?

<table>
<thead>
<tr>
<th>Score (data)</th>
<th>Percentile</th>
</tr>
</thead>
<tbody>
<tr>
<td>69</td>
<td>70</td>
</tr>
<tr>
<td>66</td>
<td>58</td>
</tr>
<tr>
<td>62</td>
<td>46</td>
</tr>
</tbody>
</table>

What % scored between 62 and 69?

<table>
<thead>
<tr>
<th>Score (data)</th>
<th>Percentile</th>
</tr>
</thead>
<tbody>
<tr>
<td>69</td>
<td>70</td>
</tr>
<tr>
<td>66</td>
<td>58</td>
</tr>
<tr>
<td>62</td>
<td>46</td>
</tr>
</tbody>
</table>

**Measures of Dispersion**

The range is the difference between the highest value and lowest value in a set of data. The standard deviation is a measure of the spread of a set of data about the mean. It can be thought of as the average of how much scores deviate from the mean.

The Normal Distribution

If data is normally distributed:
1. the mean, median, and mode are equal
2. the curve is symmetric; ½ (that is .500) of the data is above the mean, and ½ of the data is below the mean.
3. approximately 95% of the data falls within 2 standard deviations of the mean.

A z-score represents the number of standard deviations a data value is away from the mean. The z-score that corresponds to a particular data item enables the use of a table to find the percentage of the data values that lie between that data item and the mean of the distribution.

To compute a z-score for a data item, find the difference between the data item and the mean, divided by the standard deviation.
That is, for a data item, “x”, \[ z = \frac{x - \mu}{\sigma}, \]
where \( \mu \) is the mean and \( \sigma \) is the standard deviation.

**To find the percent of data under parts of the Standard Normal Curve:**

a) Percent between a given item and the mean
   Find the \( z \) value and read directly from the table

b) Percent between items on either side of the mean
   Find the \( z \) values and add the percentages obtained from the table
   (Opposite Sides Add)

c) Percent between items on the same side of the mean
   Find the \( z \) values and subtract the percentages obtained from the table
   (Same Side Subtract)

d) Percent above an item above the mean or below an item below the mean
   Find the \( z \) value of the item and subtract the percentage from .500
e) Percent above an item below the mean or below an item above the mean

Find the z value of the item and add the percentage to .500