Learning about hurricanes: Parabolas, Spirals, and Trochoides D. Quesada

## School of Science- Fechnology and Engineering Management

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UNIVERSITY
School of SCIENCE, TECHNOLOGY
\& ENGINEERING MANAGEMENT


During the time of Greeks and Romans, mythology was part of the cultural legacy left by these two cultures. The name Gaia, the goddess of planet Earth competed in beauty with Venus or Aphrodite. Many other characters were part of this celestial family of creatures.

Pre-Columbian cultures, as Mayans, Aztecs, and Caribbean Indians Tainos, developed their own body of goddess too. Onelof those was Juranken, the goddess of fury for Mayans, and Jurankán, the evil of winds for Tainos in the Caribbean. After the conquering of America by the Spanish royalty the word Jurankán evolved into what we know today Huracán.

The incursion of English and French pirates in the Caribbean probably introduced the word into the English colonies as Hurricane. According to Fernando Ortiz, a Cuban historian who wrote a book about the history of Hurricanes; there are archeological evidences that Caribbean Indians were aware of the spiral pattern today observed in hurricane rain bands. It is known also that the awareness of Tainos about hurricanes saved Columbus' fleet of a shipwreck during an incident in La Hispaniola, when the governor did not permit Columbus to be in a city.

## Born in Catalan, Spain 1837

came to Havana Cuba in 1870 as Director of the Meteorological Observatory in the Royal College of Belén.

He was a Jesuit and a Scientist. Died Havana, Cuba July 23, 1893.

through debris and questioning survivors. Everything was painstakingly recorded.
Father Viñes developed a demanding daily weather observation schedule for himself. This included observations every hour from 4 a.m. to 10 p.m., an international observation at 7:30 p.m. and, in times of special need, an observation every hour of both day and night. He recorded pressure, temperature (normal, max-


In his day, the Reverend Benito Viñes was the world's greatest authority on Atlantic hurricanes.
the storm's center, and he was also the first to forecast hurricane movement on the basis of cloud movement. The Pilot Charts of the U.S. Hydrographic Office printed and reprinted the "laws" of Father Viñes. The May issue of the 1889 Pilot Chart reads: "These important laws, established by the study and long experience of Father Vines, should be thoroughly understood by every navigator and utilized by shaping his course so as to

1875, he established Viñes Law, essential to the study of tropical storms

The May issue of the 1889 Pilot Chart reads: "These important laws, established by the study and long experience of Father Viñes, should be thoroughly understood by every navigator and utilized by shaping his course so as to avoid a hurricane."

## Father Viñes was the first

- to suggest that the clouds well in advance of a hurricane could be used to locate the storm's center
- the first to forecast hurricane movement on the basis of cloud movement
- discovered the first law of anticyclones of the Caribbean


Father Vines invented the Antilles Cyclonoscope to help other forecasters detect, locate, and track hurricanes and determine their size and intensity. The interior circle and the pointer labeled "vortice" (bottom) rotate with respect to the outer compass, showing the bearing of the eye of the storm in relation to surface wind direction and cloud movements at various levels.

On October 6, 1900, Willis L. Moore, Chief of the U.S. Weather Bureau, wrote in Colliers Weekly that ". . . probably the Reverend Benito Viñes gave more intelligent study to the investigation of tropical cyclones than any other
scientist."

## Hurricane Science - New Generations



Dr. William M. Gray and Phil Klotzbach, both from Colorado State University, head the Tropical Meteorology Project at CSU's Department of Atmospheric Sciences. Dr. Gray is a pioneer in the science of forecasting hurricanes.


Dr. Kerry Emanuel from MIT, his book "The Divine Wind" is an excellent introduction to the world of hurricanes.

Hurricane's Science


9 COMET Program


Rainfall intensity
light moderate heavy

HOW HURRICANES FORM

## (4) Crclone

Rising aird dries as it loses molsture and gans energy, Some of
the air dives backinto the gye and into bands between adjathe air dives back into the eye and into bands between adjacent thunderstorms; the rest spirals out and descends many kilometers away. Oyclones can increase ocean evaporation,
and thus cooling, by an order of magnitude as compared with and thus cooling by an order of magnede as compared witi-
normal trade winds and mix the top layess of the wate, prononima trade wolling anet cooling sea-surface temperatures by as much as five degrees Celstus (nine degrees Fahrenheit).
(3) THUNDERSTORM
An atmospheric disturbance sweeping through
$\begin{aligned} & \text { the area helps to spawn a low-pressure } 20 n e \\ & \text { (1) at the sea surface which draws in add. }\end{aligned}$
(d) at the sea surface which draws in addi-
form a system that begins to circulate because
of Coriolis forces created by the earth's sota-
$\begin{aligned} & \text { tion. Converging high winds, coupled with ait }\end{aligned}$
flowing out the top, establish a strong low-
pressure hub-the eye.


## A Simple Model of the Anatomy of a Hurricane

How tall is a hurricane?
Let keep in mind that most of the clouds and storms will be located within the troposphere. Thus, in a first approximation the top edge for a hurricane might be considered the upper boundary of the troposphere, a number around 10 km. Therefore, hurricanes are more or less 10 km tall.
How wide they are?
Typical hurricanes are considered those for which the distance from their center of circulation to their outermost closed isobar ranges between 3 and 6 latitude degrees, in other words, hurricanes with a radius between 333 and 666 kilometers (look at these numbers, interesting!!!).
With this information in mind, which picture illustrates better this situation?
The eye is normally circular in shape, and may range in size from 3 to $\mathbf{3 7 0} \mathrm{km}$ (2-230 miles) in diameter.


$A=\pi\left(R^{2}-r^{2}\right.$ Area covered by rain bands

Size of a hurricane


## A Simple Model of the Anatomy of a Hurricane

Hurricane sizes are estimated by measuring the distance from their center of circulation to their outermost closed isobar.
If the radius ( $R$ ) is:

- Less than two degrees latitude, then the cyclone is "very small" or a midget.
- Radii of 2 - 3 degrees are considered "small". Radii between 3 and 6 latitude degrees are considered "average size".
- Tropical cyclones are considered "large" when the closed isobar radius is $6-8$ degrees of latitude.
- "Very large" tropical cyclones have a radius of greater than 8 degrees.


| Hurricane size <br> classification | Radius in Degrees <br> Latitude | Radius in Km |
| :---: | :---: | :---: |
| Very Small or midget | Less than 2 |  |
| Small | Between 2 and 3 |  |
| Average size | Between 3 and 6 |  |
| Large | Between 6 and 8 |  |
| Very Large | Greater than 8 |  |



The full circumference equals $360^{\circ}$. In order to convert degrees into units of distance a simple proportion is used:
$\frac{\text { Circumference }}{360^{\circ}}=\frac{2 \pi R}{360^{\circ}}=\frac{d(A, B)}{a}$

## A Simple Model of the Anatomy of a Hurricane

Hurricanes in a first approximation can be seen as hollow cylinders, where the inner core would represent the so called eye of the hurricane, and the outer cylindrical shell contains all rain bands. By using this simple model of a hurricane,

1. Find the volume occupied by rain bands (Hint: the volume of a cylinder is given by $V=\pi R^{2} H$, where $R$ is the radius, and H the height). For typical hurricanes (average size), the size of the inner core (eye of the hurricane) is around 50 mi .
2. By using the table provided in the Introduction, find out the approximate values for volumes of water contained in all of these three basins: Pacific Ocean, Atlantic Ocean, Caribbean and Gulf of Mexico. Once you have gotten these numbers, compare them with the volume occupied by rain bands found in the previous point (a). How many hurricanes these sizes are needed to fill out each one of these basins?

|  | Average | Depth | Area |  | Volume |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Ocean or Sea | $\mathbf{m}$ | $\mathbf{f t}$ | $\mathbf{1 0}^{\mathbf{6}} \mathbf{k m}^{\mathbf{2}}$ | $\mathbf{1 0}^{\mathbf{6}} \mathbf{m i}^{\mathbf{2}}$ | $\mathbf{1 0}^{\mathbf{6}} \mathbf{k m}^{\mathbf{3}}$ | $\mathbf{1 0}^{\mathbf{6}} \mathbf{m i}^{\mathbf{3}}$ |
| Pacific Ocean | 4,300 | 14,000 | 155.6 | 60.1 | 679.6 | 163.1 |
| Atlantic Ocean | 3,900 | 12,900 | 76.8 | 30 | 313.4 | 75.2 |
| Indian Ocean | 3,900 | 12,800 | 68.5 | 26.5 | 269.3 | 64.7 |
| Gulf of Mexico and Caribbean Sea | 2,200 | 7,300 | 4.3 | 1.7 | 9.6 | 2.3 |
| Artic Ocean | 1,300 | 4,300 | 14.1 | 5.4 | 17 | 4.1 |
| Hudson Bay | 101 | 331 | 1.2 | 0.5 | 0.16 | 0.04 |
| Baltic Sea | 55 | 180 | 0.4 | 0.2 | 0.02 | 0.005 |
| Mediterranean and Black Sea | 1430 | 4,690 | 3.0 | 1.1 | 4.2 | 1 |
| World Ocean | 3790 | 12,430 | 361.1 | 139.4 | 1,370 | 329 |

$A=\pi\left(R^{2}-r^{2}\right)$
Area covered
by rain bands


## Energy transport

| 20 km | Polar Cell | Ferrell Cell | Hadley Cell <br> sub-tropical <br> jet | stratosphere <br> troposphere |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |
|  | $90^{\circ}$ easte polar high | Subpolar low LATI | $30^{\circ}$ easterly tra subtropical high |  |


http://www.crh.noaa.gov/mqt/webpics/weatherpics/Big/thunderstorm\ 1.jpg http://www.onr.navy.mil/focus/ocean/motion/currents1.htm

## Saffir-Simpson scale Inequalities at work - Saving life of people!!!

| Category | Central Pressure $\mathbf{m b}$ ( inches of $\mathbf{H g}$ ) |  | Wind Speed mp/h | Storm Surge ft (m) | Damage |
| :---: | :---: | :---: | :---: | :---: | :---: |
| TD | > 990 | ( $>29.25$ ) | $<39$ |  |  |
| TS | > 985 | ( $>29.04$ ) | 39-73 |  |  |
| H 1 | > 980 | ( > 28.94 ) | 74-95 | 4-5 (~1.5) | Damage mainly trees, shrubbery, and unanchored mobile homes |
| H 2 | 965-980 | 8.50-28.93) | 96-110 | 6-8(2-2.5) | Some trees blown down; major damage to exposed mobile homes; some damage to roofs of buildings |
| H 3 | 945-965 | 27.91-28.49) | 111-130 | 9-12 (2.5-4) | Foliage removed from trees, large trees blown down; mobile homes destroyed; some structural damage to small buildings |
| H 4 | 920-945 | (27.17-27.90) | 131-155 | 13-18(4-5.5) | All signs blown down; extensive damage to roofs; complete destruction of mobile homes; flooding inland as far as $10 \mathrm{~km}(6 \mathrm{mi})$; major damage to lower floors of structures near shore |
| H 5 | < 920 | (<27.17) | > 155 | > 18 ( > 5.5) | Severe damage to windows and doors; extensive damage to roofs of homes and industrial buildings; small buildings overturned and blown away; major damage to lower floors of all structures less than 4.5 m (15ft) above sea level within 500 m of shore |

## Wind Hazards



## Variation of the barometric pressure during as a function of time for a moving hurricane






## FLEEING HURRICANE GEORGES IN KEY WEST



## How to estimate the force exerted by the mass of water in the storm surge?



Depth of the storm surge area $=d$
The net force produced by
this Mass of water equals: $F=\frac{1}{2} \rho g w h^{2}$
 of sea water - v

Direction of motion of the mass of water

Height above sea level = h
Volume of the mass of sea water $=\mathbf{V}$

$$
V=w d h
$$

Urban areas located near the coast will Suffer the impact of the storm surge forces. If building are not made such that, they may afford a high load on their walls, then, they will collapse after the impact.

## Sea waves and its mathematical representation: Trigonometric functions at work



Hawaiian Wave


Tsunami Wave


## Building a family of trigonometric functions



Traveling Wave


Standing Wave







Figure 2 - How simple waves add together to form a random sea.
Note: Wave heighthength ratios are greatly exaggerated for clarity

Conditions Necessary for a Fully Developed Sea at Given Winds Speeds

| Wind Conditions |  |  | Wave Size |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Wind Speed in One Direction | Fetch | Wind Duration | Average Height | Average Wavelength | Average Period |
| 19 km/hr <br> $12 \mathrm{mi} / \mathrm{hr}$ | 19 km <br> 12 mi | 2 hr | $\begin{gathered} 0.27 \mathrm{~m} \\ 0.9 \mathrm{ft} \end{gathered}$ | $\begin{gathered} 8.5 \mathrm{~m} \\ 28 \mathrm{ft} \end{gathered}$ | 3.0 sec |
| 37 km/hr $23 \mathrm{mi} / \mathrm{hr}$ | $\begin{gathered} 139 \mathrm{~km} \\ 86 \mathrm{mi} \end{gathered}$ | 10 hr | $\begin{aligned} & 1.5 \mathrm{~m} \\ & 4.9 \mathrm{ft} \end{aligned}$ | $\begin{gathered} 33.8 \mathrm{~m} \\ 111 \mathrm{ft} \end{gathered}$ | 5.7 sec |
| 56 km/hr $35 \mathrm{mi} / \mathrm{hr}$ | $\begin{gathered} 518 \mathrm{~km} \\ 322 \mathrm{mi} \end{gathered}$ | 23 hr | $\begin{gathered} 4.1 \mathrm{~m} \\ 13.6 \mathrm{ft} \end{gathered}$ | $\begin{gathered} 76.5 \mathrm{~m} \\ 251 \mathrm{ft} \end{gathered}$ | 8.6 sec |
| 74 km/hr $46 \mathrm{mi} / \mathrm{hr}$ | $\begin{gathered} 1313 \mathrm{~km} \\ 816 \mathrm{mi} \end{gathered}$ | 42 hr | $\begin{gathered} 8.5 \mathrm{~m} \\ 27.9 \mathrm{ft} \end{gathered}$ | 136 m <br> 446 ft | 11.4 sec |
| 92 km/hr <br> $58 \mathrm{mi} / \mathrm{hr}$ | $\begin{gathered} 2627 \mathrm{~km} \\ 1633 \mathrm{mi} \end{gathered}$ | 69 hr | 14.8 m <br> 48.7 ft | $\begin{gathered} 212.2 \mathrm{~m} \\ 696 \mathrm{ft} \end{gathered}$ | 14.3 sec |



## Storm wave trains in Havana, Cuba due to Wilma)



## Atlantic Hurricane Basin Climatology: Mean, Median, Mode



## Global Tropical Cyclones



## Average: 84 Per Year World Wide

## ATLANTIC MAJOR HURRICANES (1944-2004)



## Ocean impacts

## Atmosphere impacts

The hurricane brings colder ocean water into the box (both from below, and from upstream)

Before


Temperature Anomaly (F) at 30,000 feet during Hurricane Katrina August 21-25, 2005


Example: The week after hurricane Gloria (1985) impacted New England, it arrived as an intense fall storm in Europe. The $4^{\text {th }}$ of October, 1985 was the warmest day in the $20^{\text {th }}$ century in Switzerland (NHC)

Imagery courtesy of GSFC/NASA:
http://svs.gsfc.nasa.gov/vis/a000000/a001000/a001066/

Causes of Global Surface Temperature Variability


- 1910's through 1930's: decreasing volcanic activity and increasing greenhouse gases contributed to a warming trend
- 1940's through 1970's: as industrial activity increased, sun-blocking sulfates and other aerosols from both volcanic and anthropogenic sources contributed to a slight cooling
- Since 1980: the increase in anthropogenic greenhouse gas emissions has overwhelmed the aerosol effect to produce global warming



How far Global Climate Changes may
affect the genesis and the dynamics of
hurricanes worldwide?


# Climatological Tracks of 

 Atlantic Basin Tropical Cyclones Most of the Hurricanes follow a parabolic path that can

## Tracking a Hurricane: Domain and Range



A point with coordinates ( $x, y$ ) where $X$ is the Longitude and $Y$ the Latitude The domain of the hurricane's track Is the collection of all longitudes, while The range is the collection of latitudes.

## Hurricane's Tracks



Possible tracks:

- A line - $y=m x+b$
- A parabola - $x=a y^{2}+b y+c$
- A cubic curve
- A curve with a loop

Is there any correlation between the kind of a curve a hurricane's track will follow and the weather or season?




## Climatology of Hurricane Tracks_Month_by_Month



## Hurricane Data Base

| Month / Track | Line | Parabola | Cubic | Other |
| :---: | :--- | :--- | :--- | :--- |
| June |  |  |  |  |
| July |  |  |  |  |
| August |  |  |  |  |
| September |  |  |  |  |
| October |  |  |  |  |
| November |  |  |  |  |

Work in teams and follow the steps

1. Pick a month
2. Go to http://www.aoml.noaa.gov/hrd/hurdat/DataByYearandStorm.htm and analyze the latest 30 years for every month. Count how many times in a Month the track is either a line, or a parabola, or a cubic function or something else.
3. Create a histogram summarizing your findings. Try to explain your results.



Total number of hurricane strikes by counties/parishes, 1900-2007
Data from NWS NHC 46: Hurricane Experience Levels of Coastal County Populations from Texas to Maine. Jerry D. Jarrell, Paul J. Hebert, and Max Mayfield. August, 1992, with updates.

Graphical Representation in a plane

$X$ intercept of a line
$\mathbf{Y}=\mathbf{m X} \mathbf{X} \mathbf{b}$ Equation of a line in the slope-intercept form m - slope or rate of change, $\mathrm{m}>0$ line goes up, $\mathrm{m}<0$ goes down $b-y$ intercept of a line
Larger the value of " $m$ " closer to the $y$-axis a line is located $\mathbf{Y}=\mathbf{m X}$ is called a linear variation or proportion $\mathbf{Y}=\mathbf{m} \mathbf{X}$ is called a linear variation or proportion
$\mathbf{Y}=\mathbf{m} / \mathbf{X}$ is called an inverse variation or inversely proportional

$\mathbf{Y}=\mathbf{a} \mathbf{X}^{2}+\mathbf{b} \mathbf{X}+\mathbf{c} \quad$ Equation of a parabola $X_{v}=-b / 2 a$ coordinate of the vertex
If a $>0$ it opens up, a < 0 it opens down
Larger the value of "a" narrower a parabola looks like

$$
m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}=\frac{\text { rise }}{\text { run }} \quad \begin{array}{r}
\text { What do variables } \mathrm{X} \text { and } \mathrm{Y} \text { have in common with the real world? } \\
\text { Cause and Effect Principle } \\
\text { X plays the role of cause or control parameter } \\
\text { Y plays the role of effect or result obtained by changing the control parameter }
\end{array}
$$



Points of the parabola $X=a Y^{2}+b Y+c$

1. Substitute $(X, Y)$ into the equation of the Parabola.
2. Set the system of linear equations:

$$
\begin{aligned}
& X_{1}=a\left(Y_{1}\right)^{2}+b Y_{1}+c \\
& X_{2}=a\left(Y_{2}\right)^{2}+b Y_{2}+c \\
& X_{3}=a\left(Y_{3}\right)^{2}+b Y_{3}+c
\end{aligned}
$$

3. Solve the system in order to obtain the coefficients $\mathrm{a}, \mathrm{b}$, and c .

$$
\begin{aligned}
& A=\operatorname{Det}\left(\begin{array}{lll}
\left(Y_{1}\right)^{2} & Y_{1} & 1 \\
\left(Y_{2}\right)^{2} & Y_{2} & 1 \\
\left(Y_{3}\right)^{2} & Y_{3} & 1
\end{array}\right) \\
& A_{1}=\operatorname{Det}\left(\begin{array}{lll}
X_{1} & Y_{1} & 1 \\
X_{2} & Y_{2} & 1 \\
X_{3} & Y_{3} & 1
\end{array}\right) \quad a=\frac{A_{1}}{A}
\end{aligned}
$$

4. Once coefficients are obtained, then find the vertex according to:

$$
Y_{V}=\frac{b^{2}}{2 a} \quad X_{V}=a\left(Y_{V}\right)^{2}+b Y_{V}+c
$$

## Polar Coordinates and Planar Curves


$X=r(\theta) \cos (\theta)$
$Y=r(\theta) \sin (\theta)$
$(x, y)$ rectangular coordinates




## Hurricanes move following a trochoidal curve



Waves are formed in a generating area where
 the wind begins to disturb

$$
\left\{\begin{array}{l}
x=a \phi-b \sin \phi \\
y=a-b \cos \phi
\end{array}\right.
$$



## Spiral patterns in hurricanes



Logarithmic Spirals - The preferred spirals of Nature


ST.THOMAS
School of SCIENCE, TECHNOLOGY
\& ENGINEERING MANAGEMENT


## BS in Mathematics

## PREREQUISITE REQUIRED COURSES: 19

## credits

MAT 205 Applied Statistics (3 credits)
MAT 232 Calculus I (4 credits)
MAT 233 Calculus II ( 4 credits)
CHE 101/L General Chemistry I + Laboratory (4 credits)
CHE 102/L General Chemistry II + Laboratory (4 credits) MAJOR REQUIREMENTS: 35 credits total
Core Mathematics Courses: ( 13 credits)
MAT 234 Calculus III (4 credits)
MAT 306 Differential Equations ( 3 credits) MAT 311 Linear Algebra (3 credits)
MAT 316 Complex Variables ( 3 credits)
Mathematics Electives: (6 credits)
Take two mathematics courses at the 300 or 400 level.
Computing Requirement: ( 6 credits)
Take two courses.
CIS 230 Introduction to Java Programming ( 3 credits)
CIS 235 Introduction to C++ Programming (3 credits)
CIS 302 Advanced C++ Programming (3 credits)
CIS 310 Advanced Java Programming ( 3 credits)
CIS 360 Data Structures ( 3 credits)
CIS 351 Systems Analysis and Design ( 3 credits)
CIS 430 Database Management Systems ( 3 credits)
Physical Science Requirements: ( 10 credits)
PHY 207/L University Physics I + Laboratory (5 credits)
PHY 208/L University Physics II + Laboratory (5 credits)
Sub-Total Credits: 54
GENERAL EDUCATION REQUIREMENTS: 42 credits
(Program requirements will satisfy 9 credits of the GER.) GENERAL ELECTIVES: 24 credits
Total credits: 120

## Mathematics and Atmospheric Sciences

 Ongoing research project \# 1: The effect of Climate and Weather Variability on Hurricane Dynamics

## Ongoing research project \# 2: Asthma - Weather connection

## Air Quality and Respiratory disorders: Modeling asthma attacks considering the environmental triggers, the mechanics of lung functioning, immune response and genetic factors.

Asthma Statistics Worldwide: A brief overview \# of people diagnosed: more than 150 M
Europe: the \# of cases has doubled
USA: the \# of cases has increased more than 60\%
India: between 15 and 20 M
Africa: between 11 and $18 \%$ population
\# of deaths yearly: around 180,000
Miami Dade County - 7.1\% Middle and HS ch were reported with a $\qquad$
The \# of hospitalizations due to asthma has doubled.
The \# 1 cause of school absences and $35 \%$ of parents missed work



Pet dander

Asthma Prevalence by Age United States, 1980-1994


## Urban Heat Island Effect

Man is likely playing a role in climate change through urbanization and land use changes competing with greenhouse Gases and cycles of Nature

- In cities, vertical walls, steel and concrete absorb the sun's heat and are slow to cool at night
- Nights may be 10 or more degrees warmer in and near cities than in rural areas some nights
- Temperatures measured in cities increase as they grow. Asthma-related Absences, in Millions, 2002-03



Leaders Make the Discoveries That Build Our Future

## Science \& Mathematics Fellows Program



- Start Date: August 2008-30 freshmen \& 30 juniors who transfer with an AA.
- Qualified students may receive financial aid and academic scholarships.
- Research based in state of the art Science \& Technology facility.



