1. **(E)** Suppose the two numbers are a and b. Then the desired sum is

$$2(a+3)+2(b+3)=2(a+b)+12=2S+12.$$

2. **(E)** Suppose $N=10a+b$. Then $10a+b=ab+(a+b)$. It follows that $9a=ab$, which implies that $b=9$, since $a\neq 0$.

3. **(B)** If Kristin's annual income is $x\geq 28000$ dollars, then

$$\frac{p}{100}\cdot 28000+\frac{p+2}{100}\cdot(x-28000)=\frac{p+0.25}{100}\cdot x.$$

Multiplying by 100 and expanding yields

$$28000p+px+2x-28000p-56000=px+0.25x.$$

So, $1.75x=\frac{7}{4}x=56000$ and $x=32000$.

4. **(D)** Since the median is 5, we can write the three numbers as x, 5, and y, where

$$\frac{1}{3}(x+5+y)=x+10 \quad \text{and} \quad \frac{1}{3}(x+5+y)+15=y.$$

If we add these equations, we get

$$\frac{2}{3}(x+5+y)+15=x+y+10,$$

and solving for $x+y$ gives $x+y=25$. Hence, the sum of the numbers is $x+5+y=30$.

OR

Let m be the mean of the three numbers. Then the least of the numbers is $m-10$ and the greatest is $m+15$. The middle of the three numbers is the median, 5. So

$$\frac{1}{3}\left((m-10)+5+(m+15)\right)=m$$

and $m=10$. Hence, the sum of the three numbers is $3(10)=30$.

5. **(D)** Note that

$$1 \cdot 3 \cdots 9999 = \frac{1 \cdot 2 \cdot 3 \cdots 9999 \cdot 10000}{2 \cdot 4 \cdots 10000} = \frac{10000!}{2^{5000} \cdot 1 \cdot 2 \cdots 5000} = \frac{10000!}{2^{5000} \cdot 5000!}.$$

6. **(E)** The last four digits (GHIJ) are either 9753 or 7531, and the remaining odd digit (either 1 or 9) is A, B, or C. Since A + B + C = 9, the odd digit among A, B, and C must be 1. Thus the sum of the two even digits in ABC is 8. The three digits in DEF are $864, 642$, or 420, leaving the pairs 2 and 0, 8 and 0, or 8 and 6, respectively, as the two even digits in ABC. Of those, only the pair 8 and 0 has sum 8, so ABC is 810, and the required first digit is 8. The only such telephone number is 810-642-9753.

7. **(A)** Let n be the number of full-price tickets and p be the price of each in dollars. Then

$$np + (140 - n) \cdot \frac{p}{2} = 2001, \quad \text{so} \quad p(n + 140) = 4002.$$

Thus $n + 140$ must be a factor of $4002 = 2 \cdot 3 \cdot 23 \cdot 29$. Since $0 \le n \le 140$, we have $140 \le n + 140 \le 280$, and the only factor of 4002 that is in the required range for $n + 140$ is $174 = 2 \cdot 3 \cdot 29$. Therefore, $n + 140 = 174$, so $n = 34$ and $p = 23$. The money raised by the full-price tickets is $34 \cdot 23 = 782$ dollars.

8. **(C)** The slant height of the cone is 10, the radius of the sector. The circumference of the base of the cone is the same as the length of the sector's arc. This is $252/360 = 7/10$ of the circumference, 20π, of the circle from which the sector is cut. The base circumference of the cone is 14π, so its radius is 7.

9. **(C)** Note that

$$f(600) = f\left(500 \cdot \frac{6}{5}\right) = \frac{f(500)}{6/5} = \frac{3}{6/5} = \frac{5}{2}.$$

OR

For all positive x,

$$f(x) = f(1 \cdot x) = \frac{f(1)}{x},$$

so $xf(x)$ is the constant $f(1)$. Therefore,

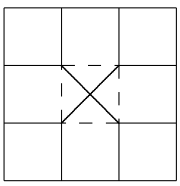
$$600f(600) = 500f(500) = 500(3) = 1500,$$

so $f(600) = \frac{1500}{600} = \frac{5}{2}$.

Note. $f(x) = \frac{1500}{x}$ is the unique function satisfying the given conditions.

10. **(D)** The pattern shown below is repeated in the plane. In fact, nine repetitions of it are shown in the statement of the problem. Note that four of the nine squares in the three-by-three square are not in the four pentagons that make up the three-by-three square. Therefore, the percentage of the plane that is enclosed by pentagons is

$$1 - \frac{4}{9} = \frac{5}{9} = 55\frac{5}{9}\%.$$



11. **(D)** Think of continuing the drawing until all five chips are removed from the box. There are ten possible orderings of the colors: RRRWW, RRWRW, RWRRW, WRRRW, RRWWR, RWRWR, WRRWR, RWWRR, WRWRR, and WWRRR. The six orderings that end in R represent drawings that would have ended when the second white chip was drawn.

OR

Imagine drawing until only one chip remains. If the remaining chip is red, then that draw would have ended when the second white chip was removed. The last chip will be red with probability 3/5.

12. **(B)** For integers not exceeding 2001, there are $\lfloor 2001/3 \rfloor = 667$ multiples of 3 and $\lfloor 2001/4 \rfloor = 500$ multiples of 4. The total, 1167, counts the $\lfloor 2001/12 \rfloor = 166$ multiples of 12 twice, so there are $1167 - 166 = 1001$ multiples of 3 or 4. From these we exclude the $\lfloor 2001/15 \rfloor = 133$ multiples of 15 and the $\lfloor 2001/20 \rfloor = 100$ multiples of 20, since these are multiples of 5. However, this excludes the $\lfloor 2001/60 \rfloor = 33$ multiples of 60 twice, so we must re-include these. The number of integers satisfying the conditions is $1001-133-100+33 = 801$.

13. **(E)** The equation of the first parabola can be written in the form

$$y = a(x-h)^2 + k = ax^2 - 2axh + ah^2 + k,$$

and the equation for the second (having the same shape and vertex, but opening in the opposite direction) can be written in the form

$$y = -a(x-h)^2 + k = -ax^2 + 2axh - ah^2 + k.$$

Hence,

$$a+b+c+d+e+f = a+(-2ah)+(ah^2+k)+(-a)+(2ah)+(-ah^2+k) = 2k.$$

OR

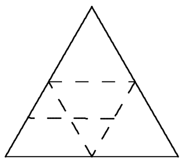
The reflection of a point (x, y) about the line $y = k$ is $(x, 2k - y)$. Thus, the equation of the reflected parabola is

$$2k - y = ax^2 + bx + c, \quad \text{or equivalently,} \quad y = 2k - (ax^2 + bx + c).$$

Hence $a + b + c + d + e + f = 2k$.

14. **(D)** Each of the $\binom{9}{2} = 36$ pairs of vertices determines two equilateral triangles, for a total of 72 triangles. However, the three triangles $A_1A_4A_7$, $A_2A_5A_8$, and $A_3A_6A_9$ are each counted 3 times, resulting in an overcount of 6. Thus, there are 66 distinct equilateral triangles.

15. **(B)** Unfold the tetrahedron onto a plane. The two opposite-edge midpoints become the midpoints of opposite sides of a rhombus with sides of length 1, so are now 1 unit apart. Folding back to a tetrahedron does not change the distance and it remains minimal.



16. **(D)** Number the spider's legs from 1 through 8, and let a_k and b_k denote the sock and shoe that will go on leg k. A possible arrangement of the socks and shoes is a permutation of the sixteen symbols $a_1, b_1, \ldots, a_8, b_8$, in which a_k precedes b_k for $1 \le k \le 8$. There are 16! permutations of the sixteen symbols, and a_1 precedes b_1 in exactly half of these, or $16!/2$ permutations. Similarly, a_2 precedes b_2 in exactly half of those, or $16!/2^2$ permutations. Continuing, we can conclude that a_k precedes b_k for $1 \le k \le 8$ in exactly $16!/2^8$ permutations.

17. **(C)** Since $\angle APB = 90^\circ$ if and only if P lies on the semicircle with center $(2,1)$ and radius $\sqrt{5}$, the angle is obtuse if and only if the point P lies inside this semicircle. The semicircle lies entirely inside the pentagon, since the distance, 3, from $(2,1)$ to $\overline{DE}$ is greater than the radius of the circle. Thus, the probability that the angle is obtuse is the ratio of the area of the semicircle to the area of the pentagon.

Let $O = (0,0)$, $A = (0,2)$, $B = (4,0)$, $C = (2\pi+1,0)$, $D = (2\pi+1,4)$, and $E = (0,4)$. Then the area of the pentagon is

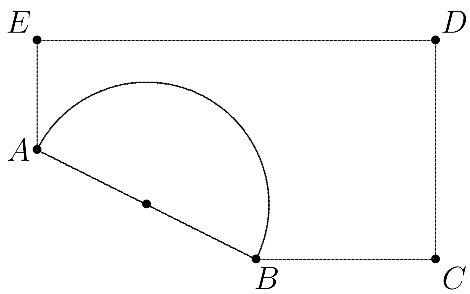
$$[ABCDE] = [OCDE] - [OAB] = 4 \cdot (2\pi + 1) - \frac{1}{2}(2 \cdot 4) = 8\pi,$$

and the area of the semicircle is

$$\frac{1}{2}\pi\left(\sqrt{5}\right)^2 = \frac{5}{2}\pi.$$

The probability is

$$\frac{\frac{5}{2}\pi}{8\pi} = \frac{5}{16}.$$



18. **(D)** Let C be the intersection of the horizontal line through A and the vertical line through B. In right triangle ABC, we have $BC = 3$ and $AB = 5$, so $AC = 4$. Let x be the radius of the third circle, and D be the center. Let E and F be the points of intersection of the horizontal line through D with the vertical lines through B and A, respectively, as shown.

In $\triangle BED$ we have $BD = 4 + x$ and $BE = 4 - x$, so

$$DE^2 = (4 + x)^2 - (4 - x)^2 = 16x,$$

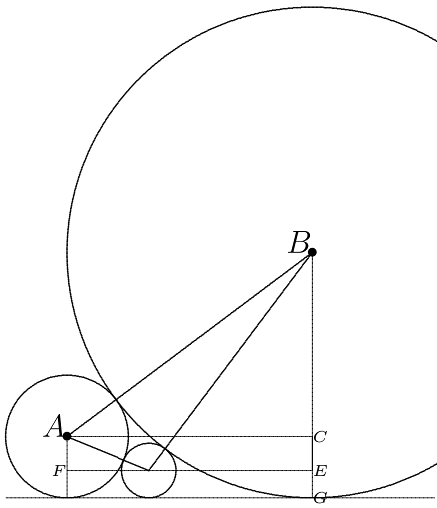
and $DE = 4\sqrt{x}$. In $\triangle ADF$ we have $AD = 1 + x$ and $AF = 1 - x$, so

$$FD^2 = (1 + x)^2 - (1 - x)^2 = 4x,$$

and $FD = 2\sqrt{x}$. Hence,

$$4 = AC = FD + DE = 2\sqrt{x} + 4\sqrt{x} = 6\sqrt{x}$$

and $\sqrt{x} = \frac{2}{3}$, which implies $x = \frac{4}{9}$.



19. **(A)** The sum and product of the zeros of $P(x)$ are $-a$ and $-c$, respectively. Therefore,

$$-\frac{a}{3} = -c = 1 + a + b + c.$$

Since $c = P(0)$ is the y-intercept of $y = P(x)$, it follows that $c = 2$. Thus $a = 6$ and $b = -11$.

20. **(C)** Let the midpoints of sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{DA}$ be denoted M, N, P, and Q, respectively. Then $M = (2, 5)$ and $N = (3, 2)$. Since $\overline{MN}$ has slope -3, the slope of $\overline{MQ}$ must be $1/3$, and $MQ = MN = \sqrt{10}$. An equation for the line containing $\overline{MQ}$ is thus $y - 5 = \frac{1}{3}(x - 2)$, or $y = (x + 13)/3$. So Q has coordinates of the form $\left(a, \frac{1}{3}(a + 13)\right)$. Since $MQ = \sqrt{10}$, we have

$$\begin{aligned} (a-2)^2 + \left(\frac{a+13}{3} - 5\right)^2 &= 10 \\ (a-2)^2 + \left(\frac{a-2}{3}\right)^2 &= 10 \\ \frac{10}{9}(a-2)^2 &= 10 \\ (a-2)^2 &= 9 \\ a - 2 &= \pm 3 \end{aligned}$$

Since Q is in the first quadrant, $a = 5$ and $Q = (5, 6)$. Since Q is the midpoint of $\overline{AD}$ and $A = (3, 9)$, we have $D = (7, 3)$, and $7 + 3 = 10$.

OR

Use translation by vectors. As before, $M = (2, 5)$ and $N = (3, 2)$. So $\overrightarrow{NM} = \langle -1, 3\rangle$. The vector $\overrightarrow{MQ}$ must have the same length as $\overrightarrow{MN}$ and be perpendicular to it, so $\overrightarrow{MQ} = \langle 3, 1\rangle$. Thus, $Q = (5, 6)$. As before, $D = (7, 3)$, and the answer is 10.

OR

Each pair of opposite sides of the square are parallel to a diagonal of $ABCD$, so the diagonals of $ABCD$ are perpendicular. Similarly, each pair of opposite sides of the square has length half that of a diagonal, so the diagonals of $ABCD$ are congruent. Since the slope of $\overline{AC}$ is -3 and $\overline{AC}$ is perpendicular to $\overline{BD}$, we have

$$\frac{b-1}{a-1} = \frac{1}{3}, \quad \text{so} \quad a - 1 = 3(b - 1).$$

Since $AC = BD$,

$$40 = (a-1)^2 + (b-1)^2 = 9(b-1)^2 + (b-1)^2 = 10(b-1)^2,$$

and since b is positive, $b = 3$ and $a = 1 + 3(b-1) = 7$. So the answer is 10.

21. **(D)** Note that

$$(a+1)(b+1) = ab + a + b + 1 = 524 + 1 = 525 = 3 \cdot 5^2 \cdot 7,$$

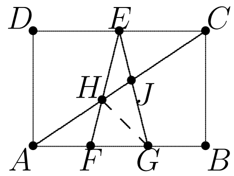
and

$$(b+1)(c+1) = bc + b + c + 1 = 146 + 1 = 147 = 3 \cdot 7^2.$$

Since $(a+1)(b+1)$ is a multiple of 25 and $(b+1)(c+1)$ is not a multiple of 5, it follows that $a+1$ must be a multiple of 25. Since $a+1$ divides 525, a is one of 24, 74, 174, or 524. Among these only 24 is a divisor of 8!, so $a = 24$. This implies that $b+1 = 21$, and $b = 20$. From this it follows that $c+1 = 7$ and $c = 6$. Finally, $(c+1)(d+1) = 105 = 3 \cdot 5 \cdot 7$, so $d+1 = 15$ and $d = 14$. Therefore, $a - d = 24 - 14 = 10$.

22. **(C)** The area of triangle EFG is $(1/6)(70) = 35/3$. Triangles AFH and CEH are similar, so $3/2 = EC/AF = EH/HF$ and $EH/EF = 3/5$. Triangles AGJ and CEJ are similar, so $3/4 = EC/AG = EJ/JG$ and $EJ/EG = 3/7$.

Since the areas of the triangles that have a common altitude are proportional to their bases, the ratio of the area of $\triangle EHJ$ to the area of $\triangle EHG$ is $3/7$, and the ratio of the area of $\triangle EHG$ to that of $\triangle EFG$ is $3/5$. Therefore, the ratio of the area of $\triangle EHJ$ to the area of $\triangle EFG$ is $(3/5)(3/7) = 9/35$. Thus, the area of $\triangle EHJ$ is $(9/35)(35/3) = 3$.



23. **(A)** If r and s are the integer zeros, the polynomial can be written in the form

$$P(x) = (x - r)(x - s)(x^2 + \alpha x + \beta).$$

The coefficient of x^3, $\alpha - (r+s)$, is an integer, so α is an integer. The coefficient of x^2, $\beta - \alpha(r + s) + rs$, is an integer, so β is also an integer. Applying the quadratic formula gives the remaining zeros as

$$\frac{1}{2}(-\alpha \pm \sqrt{\alpha^2 - 4\beta}) = -\frac{\alpha}{2} \pm i\frac{\sqrt{4\beta - \alpha^2}}{2}.$$

Answer choices **(A)**, **(B)**, **(C)**, and **(E)** require that $\alpha = -1$, which implies that the imaginary parts of the remaining zeros have the form $\pm\sqrt{4\beta - 1}/2$. This is true only for choice **(A)**.
Note that choice **(D)** is not possible since this choice requires $\alpha = -2$, which produces an imaginary part of the form $\sqrt{\beta - 1}$, which cannot be $\frac{1}{2}$.

24. **(D)** Let E be a point on $\overline{AD}$ such that $\overline{CE}$ is perpendicular to $\overline{AD}$, and draw $\overline{BE}$. Since $\angle ADC$ is an exterior angle of $\triangle ADB$, it follows that

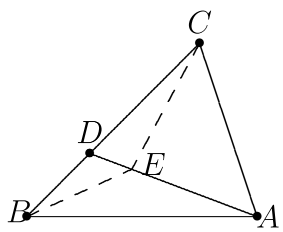
$$\angle ADC = \angle DAB + \angle ABD = 15^\circ + 45^\circ = 60^\circ.$$

Thus, $\triangle CDE$ is a 30°– 60°– 90° triangle and $DE = \frac{1}{2}CD = BD$. Hence, $\triangle BDE$ is isosceles and $\angle EBD = \angle BED = 30^\circ$. But $\angle ECB$ is also equal to 30° and therefore $\triangle BEC$ is isosceles with $BE = EC$. On the other hand,

$$\angle ABE = \angle ABD - \angle EBD = 45^\circ - 30^\circ = 15^\circ = \angle EAB.$$

Thus, $\triangle ABE$ is isosceles with $AE = BE$. Hence $AE = BE = EC$.
The right triangle AEC is also isosceles with $\angle EAC = \angle ECA = 45^\circ$. Hence,

$$\angle ACB = \angle ECA + \angle ECD = 45^\circ + 30^\circ = 75^\circ.$$



25. **(D)** If a, b, and c are three consecutive terms of such a sequence, then $ac - 1 = b$, which can be rewritten as $c = (1 + b)/a$. Applying this rule recursively and simplifying yields

$$\ldots\ , a\ ,\ b\ ,\ \frac{1+b}{a}\ ,\ \frac{1+a+b}{ab}\ ,\ \frac{1+a}{b}\ ,\ a\ ,\ b\ ,\ \ldots$$

This shows that at most five different terms can appear in such a sequence. Moreover, the value of a is determined once the value 2000 is assigned to b and the value 2001 is assigned to another of the first five terms. Thus, there are four such sequences that contain 2001 as a term, namely

2001, 2000, 1, $\frac{1}{1000}$, $\frac{1001}{1000}$, 2001, $\ldots$,

1, 2000, 2001, $\frac{1001}{1000}$, $\frac{1}{1000}$, 1, $\ldots$,

$\frac{2001}{4001999}$, 2000, 4001999, 2001, $\frac{2002}{4001999}$, $\frac{2001}{4001999}$, $\ldots$, and

4001999, 2000, $\frac{2001}{4001999}$, $\frac{2002}{4001999}$, 2001, 4001999, $\ldots$,

respectively. The four values of x are 2001, 1, $\frac{2001}{4001999}$, and 4001999.