MAC2311
Exam #1

Name_____ Grade_____

Student ID_____ Date_____

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Use the addition formulas to derive the identity.

1) $\sin\left(x - \frac{\pi}{2}\right) = -\cos x$ 1) _____

State the domain and range of the function.

2) $f(x) = \frac{3}{5 + e^x}$ 2) _____

Find the slope of the curve at the given point P and an equation of the tangent line at P.

3) $y = x^2 + 11x - 15$, P(1, –3) 3) _____

Find the limit if it exists.

4) $\lim_{x \to 25} x^{1/2}$ 4) _____

5) $\lim_{x \to -10} 8x(x + 8)(x - 4)$ 5) _____

Find the limit, if it exists.

6) $\lim\limits_{h \to 0} \dfrac{(1+h)^{1/3} - 1}{h}$

6) _____

Find the limit L for the given function f, the point c, and the positive number ε. Then find a number $\delta > 0$ such that, for all x, $0 < |x - c| < \delta \Rightarrow |f(x) - L| < \varepsilon$.

7) $f(x) = \dfrac{36}{x}$, $c = 9$, $\varepsilon = 0.2$

7) _____

Find the limit.

8) $\lim\limits_{x \to -2^-} (x + 5)\left(\dfrac{|x + 2|}{x + 2}\right)$

8) _____

Find the limit using $\lim\limits_{x=0} \dfrac{\sin x}{x} = 1$.

9) $\lim\limits_{x \to 0} \dfrac{x^2 - 2x + \sin x}{x}$

9) _____

10) $\lim_{x \to 0} \frac{x}{\sin 3x}$

10) _____

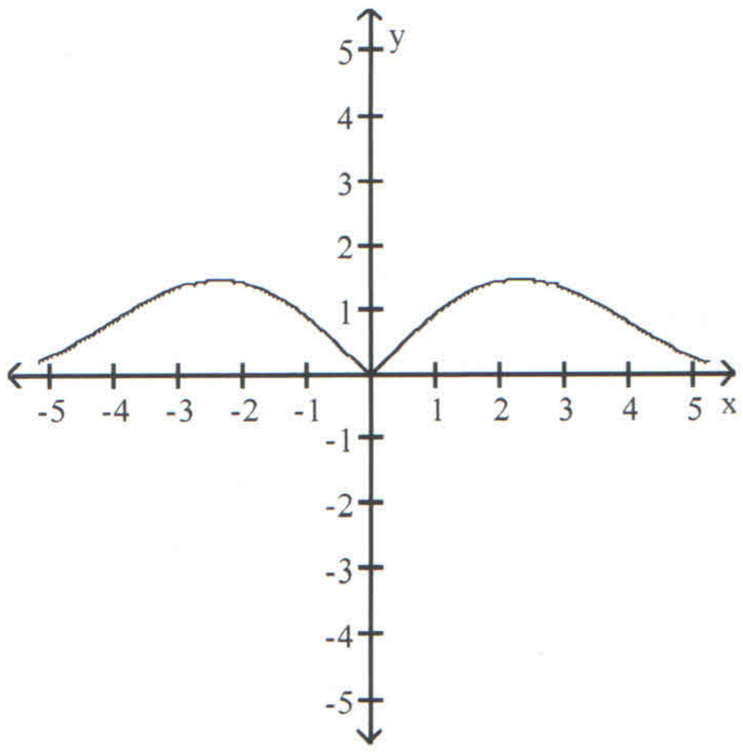
11) $\lim_{x \to 0} \frac{\sin 5x}{x}$

11) _____

Use the graph to estimate the specified limit.

12) Find $\lim_{x \to 0} f(x)$

12) _____



Determine if the given function can be extended to a continuous function at x = 0. If so, approximate the extended function's value at x = 0 (rounded to four decimal places if necessary). If not, determine whether the function can be continuously extended from the left or from the right and provide the values of the extended functions at x = 0. Otherwise write "no continuous extension."

13) $f(x) = \dfrac{\cos 2x}{|2x|}$

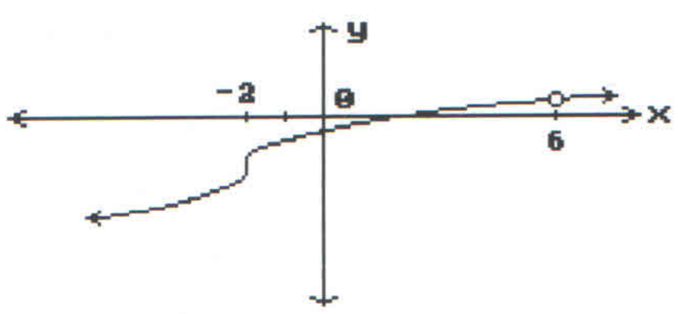
13) _____

Find the limit and determine if the function is continuous at the point being approached.

14) $\lim_{\theta \to -\pi} \tan(\sin(-\pi \cos(\sin \theta)))$ 14) _____

Find all points where the function is discontinuous.

15) 15) _____



Find the limit and determine if the function is continuous at the point being approached.

16) $\lim_{x \to 1} \cos\left(\frac{\pi}{3} \ln(e^x)\right)$ 16) _____

Provide an appropriate response.

17) Use the Intermediate Value Theorem to prove that $x(x - 3)^2 = 3$ has a solution between 2 and 4. 17) _____

Find the limit.

18) $\lim_{x \to 5^-} \frac{1}{x^2 - 25}$ 18) _____

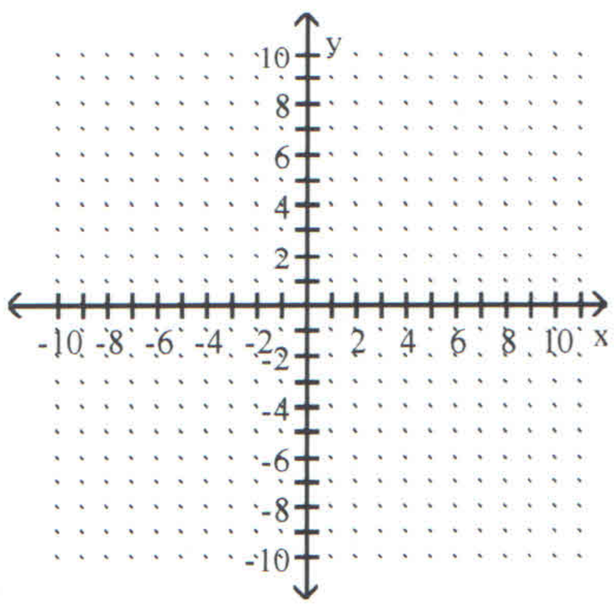
19) $\lim_{x \to 5^-} \frac{1}{x - 5}$

19) _____

Find a function that satisfies the given conditions and sketch its graph.

20) $\lim_{x \to \pm\infty} f(x) = 0$, $\lim_{x \to 4^-} f(x) = \infty$, $\lim_{x \to 4^+} f(x) = \infty$.

20) _____



Find the limit.

21) $\lim_{x \to 0^+} \left(\frac{1}{x^{1/3}} + 9 \right)$

21) _____

Answer Key
Testname:

1) $\sin\left(x - \frac{\pi}{2}\right) = \sin x \cos\left(-\frac{\pi}{2}\right) + \cos x \sin\left(-\frac{\pi}{2}\right)$
$= \sin x\,(0) + \cos x\,(-1)$
$= 0 - \cos x$
$= -\cos x$

2) domain: $(-\infty, \infty)$; range: $\left(0, \frac{3}{5}\right]$

3) slope is 13; $y = 13x - 16$

4) 5

5) -2240

6) 1/3

7) $L = 4$; $\delta = 0.47$

8) -3

9) -1

10) $\frac{1}{3}$

11) 5

12) 0

13) No continuous extension

14) 0; yes

15) $x = 6$

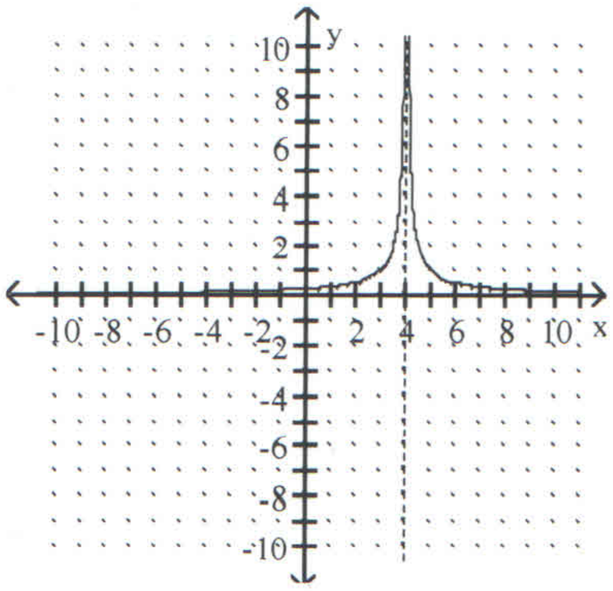
16) $\frac{1}{2}$; yes

17) Let $f(x) = x(x - 3)^2$ and let $y_0 = 3$. $f(2) = 2$ and $f(4) = 4$. Since f is continuous on $[2, 4]$ and since $y_0 = 3$ is between $f(2)$ and $f(4)$, by the Intermediate Value Theorem, there exists a c in the interval $(2, 4)$ with the property that $f(c) = 3$. Such a c is a solution to the equation $x(x - 3)^2 = 3$.

18) $-\infty$

19) $-\infty$

20) (Answers may vary.) Possible answer: $f(x) = \frac{1}{|x - 4|}$.



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